A (Zernike) moment-based nonlocal-means algorithm for image denoising

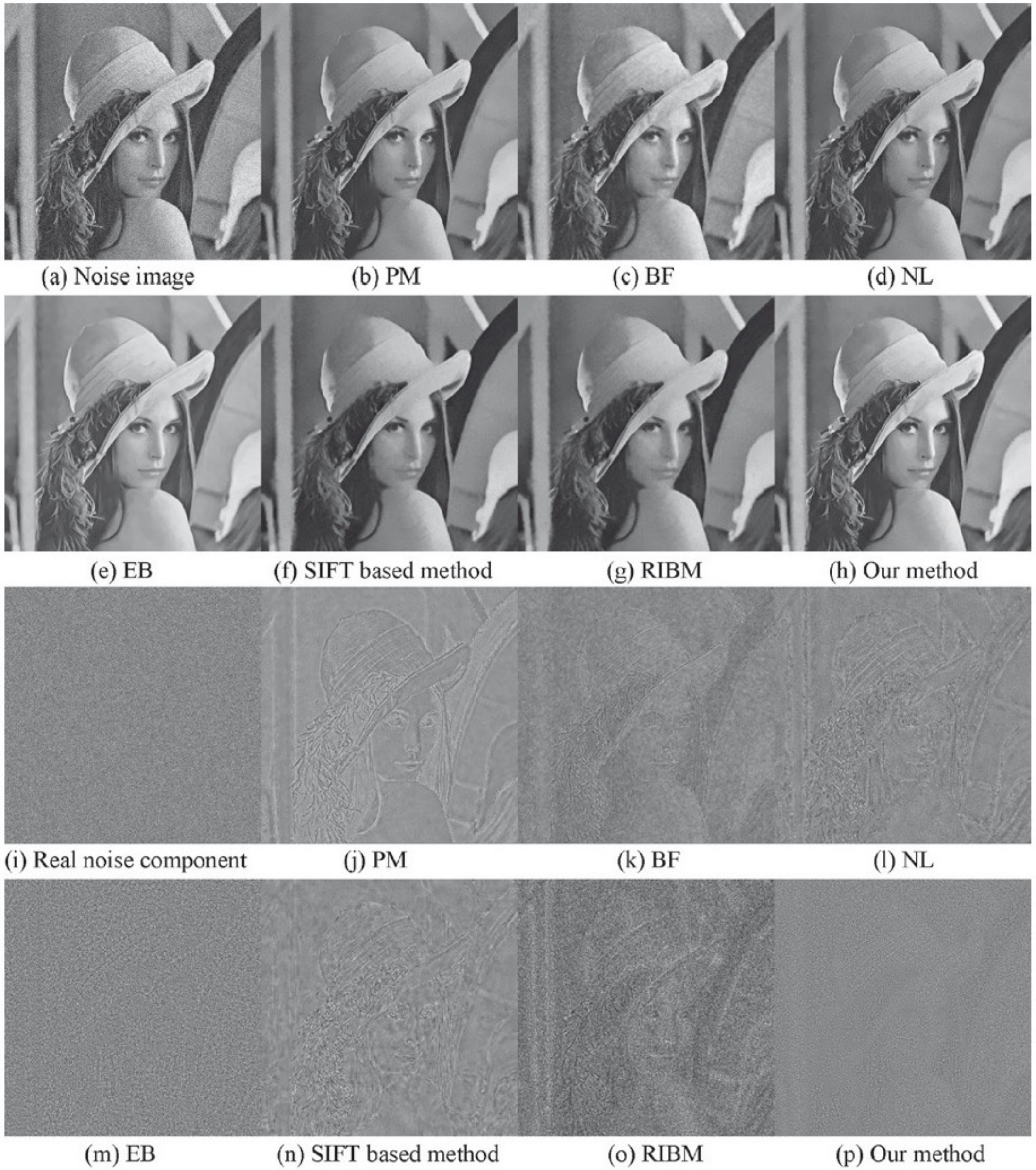
Michal Kuneš
xkunes@utia.cas.cz

ZOI UTIA, ASCR, Friday seminar 13.03.2015

Introduction

- Uses nonlocal (NL) means filter
- Introduce the Zernike moments (rotation invariant)
- Zernike moments in small local windows of each pixel are computed (local structure information)
- similarities are computed (insted of pixel intensity)
- it can gat much more pixels with higher similarity measure

Zexuan Ji, Qiang Chen, Quan-Sen Sun, and De-Shen Xia: **A moment-based nonlocal-means algorithm for image denoising**. Inf. Process. Lett. 109, 23-24 (November 2009), 1238-1244. DOI=10.1016/j.ipl.2009.09.007 http://dx.doi.org/10.1016/j.ipl.2009.09.007



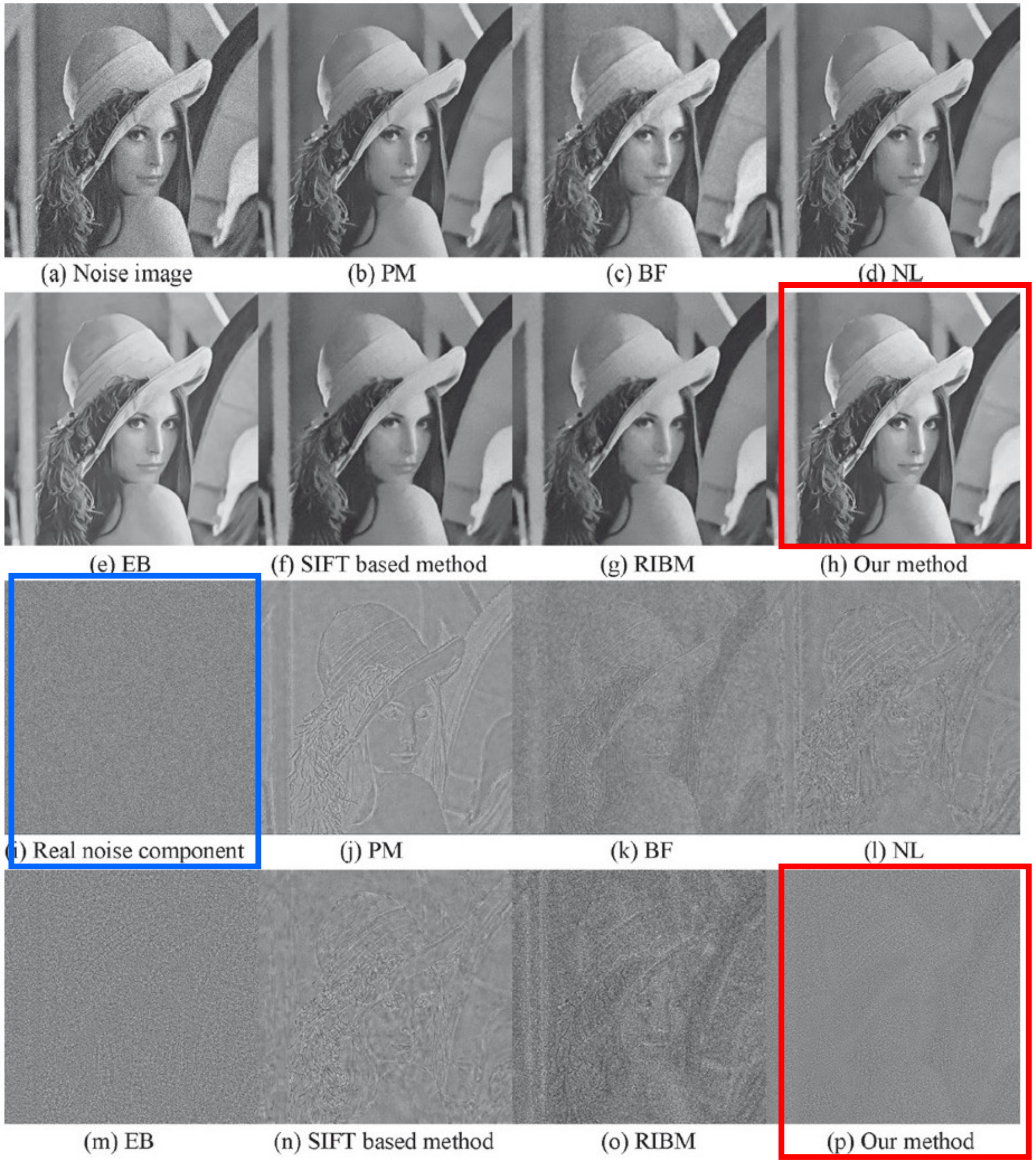
a) Noise $\sigma = 20$ (PSNR = 22.16)
b) PM model (28.83)
c) Bilateral f. (29.16)
e) NL-means (31.09)
e) Exemplar-based method (**32.64**)
f) SIFT based m. (31.26)
g) rotationally invariant block matching (31.75)
h) **Moment base NL-means** (**32.29**)

(blockmatching and 3D f. (***33.05***))

i) **real noise component**

j) – p) corresponding noise component of each method

$$PSNR = 10 \log_{10} \frac{255^2}{\sum_{i \in I} \left(NL(u)(i) - u_0(i) \right)^2 / |I|} [dB]$$



a) Noise σ = 20 (PSNR = 22.16)
b) PM model (28.83)
c) Bilateral f. (29.16)
e) NL-means (31.09)
e) Exemplar-based method (**32.64**)
f) SIFT based m. (31.26)
g) rotationally invariant block matching (31.75)
h) **Moment base NL-means** (**32.29**)

*(blockmatching and 3D f. (**33.05**))*

i) **real noise component**

j) – p) corresponding noise component of each method

$$PSNR = 10\log_{10}\frac{255^2}{\sum_{i\in I}\left(NL(u)(i)-u_0(i)\right)^2/|I|}[dB]$$

NL-means filter

$$NL(u)(i) = \sum_{j \in I} \omega(i,j) u(j)$$

$$\omega(i,j) = \frac{1}{C(i)} e^{-\frac{\left\| u(N_i) - u(N_j) \right\|_{G_\rho}^2}{h^2}} \qquad C(i) = \sum_j e^{-\frac{\left\| u(N_i) - u(N_j) \right\|_{G_\rho}^2}{h^2}}$$

$$0 \le \omega(i,j) \le 1 \qquad \sum_j \omega(i,j) = 1$$

$u(j)$ – intensity value

$w(i,j)$ – weight, depends on the similarity between pixels i and j

$u(N_i)$ – intensity gray level vector

N_i – square neighborhood of fixed size and centered at a pixel i

G_ρ – Gauss kernel with standard deviation ρ. $\left\| u(N_i) - u(N_j) \right\|_{G_\rho}^2 = G_\rho * \left\| u(N_i) - u(N_j) \right\|^2$

$C(i)$ – normalizing konstant

h – degree of filtering

NL-means filter + Moments

NL-means:

- improves image quality
- high computational cost
- similarity of patches is only translation invariant

Zimmer et al. uses the **Hu moments**

+ common, simplest
- not efficient for image features representation
- certain degree of information redundancy

S. Zimmer, S. Didas, J. Weickert, A rotationally invariant block matching strategy improving image denoising with non-local means, in: Proc. 2008 Int. Workshop on Local and Non-Local Approximation in Image Processing, in: LNLA, vol. 2008, 2008.

-> **Zernike moments**

- global shape descriptors
- particulary robust

Main points

- compute Zernike moments within a small window around each pixel
- adds orientation invariants for pixels with similarity
- removes the Gauss kernel used in NL-means algorithm
- every moment has equal possibility to influence the brightness of the central pixel

- Result: higher signal-to-noise ratio (on synthetic images)

Zernike polynomials / moments

- mathematical simplicity and universality
- set of orthogonal basis functions mapped over the unit circle

Main properties:
- orthogonality
- rotation invariance
- information compaction

$$Z_{pq} = \frac{p+1}{\pi} \iint_{x^2+y^2 \leq 1} V_{pq}^{*}(x, y) f(x, y) \, dx dy$$

p – order

q – repetition

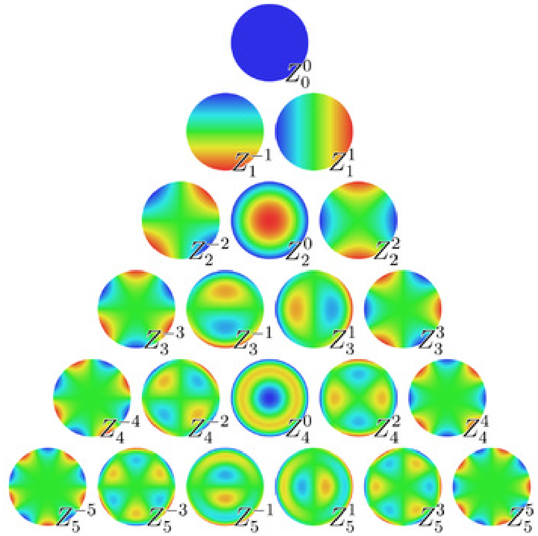
$$D = (p, q); 0 \leq p \leq \infty, |q| \leq p, |p-q| = even$$

Zernike polynomials / moments

$$Z_{pq} = \frac{p+1}{\pi} \iint\limits_{x^2+y^2 \le 1} V_{pq}^{*}(x,y)\, f(x,y)\, dx dy$$

$$Z_{pq} = \frac{p+1}{\pi} \sum_{x} \sum_{y} V_{pq}^{*}(x,y)\, f(x,y); \quad x^2 + y^2 \le 1$$

$$D = (p,q);\, 0 \le p \le \infty,\, |q| \le p,\, |p-q| = even$$



$$V_{pq}(\rho,\theta) = R_{pq}(\rho)\, e^{iq\theta}$$

$$R_{pq}(\rho) = \sum_{\substack{k=|q| \\ |p-k|=even}}^{p} \frac{(-1)^{\frac{p-k}{2}} \frac{p+k}{2}!}{\frac{p-k}{2}!\, \frac{k-q}{2}!\, \frac{k+q}{2}!} \rho^{k}$$

p – order

q – repetition

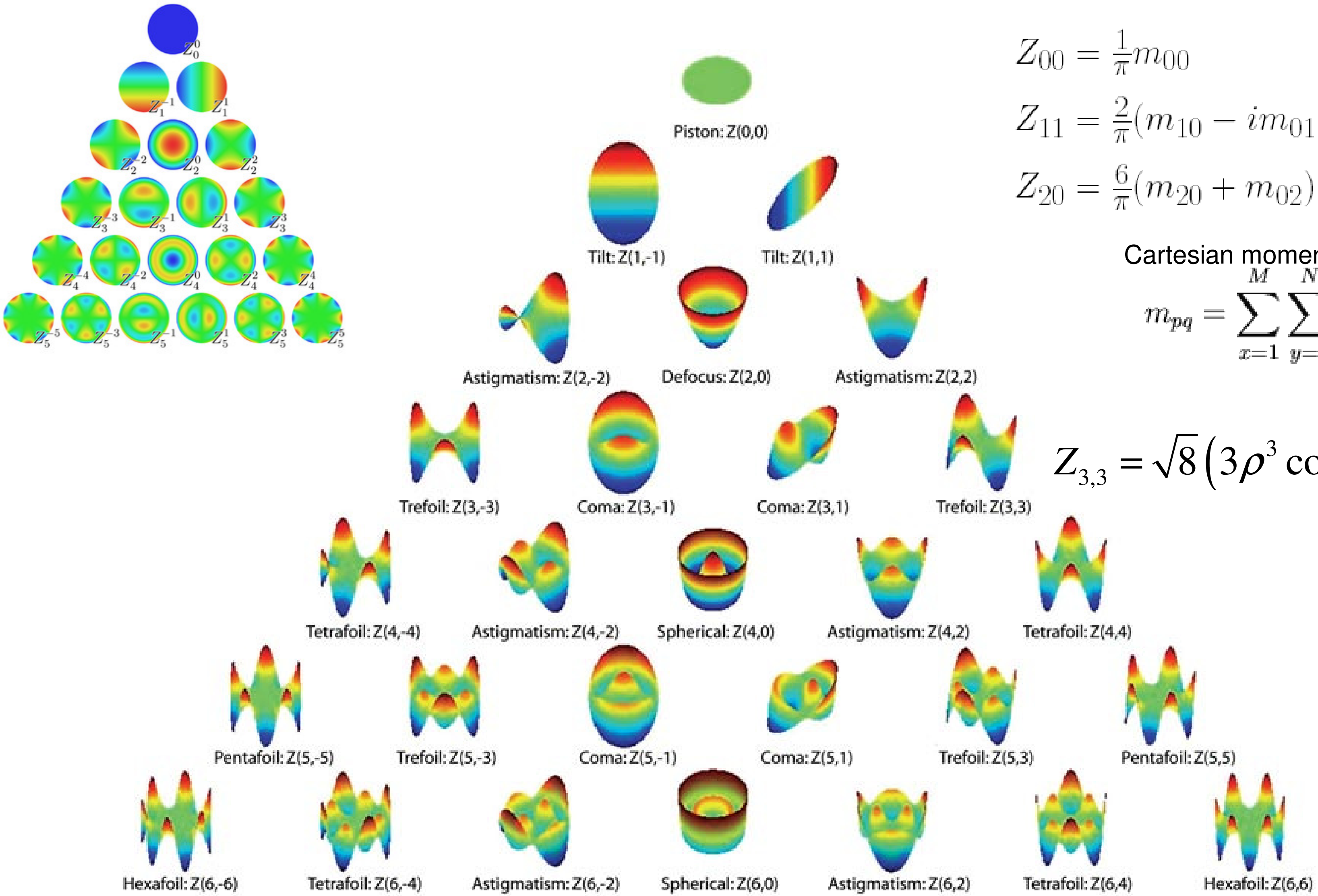
V_{pq}^{*} – complex conjugate of V_{pq}

R_{pq} – radial polynomial

ρ – length of vector from origin to pixel (x,y)

θ – angle of ρ from x axis

Zernike polynomials / moments



$$Z_{00} = \frac{1}{\pi} m_{00}$$

$$Z_{11} = \frac{2}{\pi}(m_{10} - i m_{01})$$

$$Z_{20} = \frac{6}{\pi}(m_{20} + m_{02}) - \frac{3}{\pi} m_{00}$$

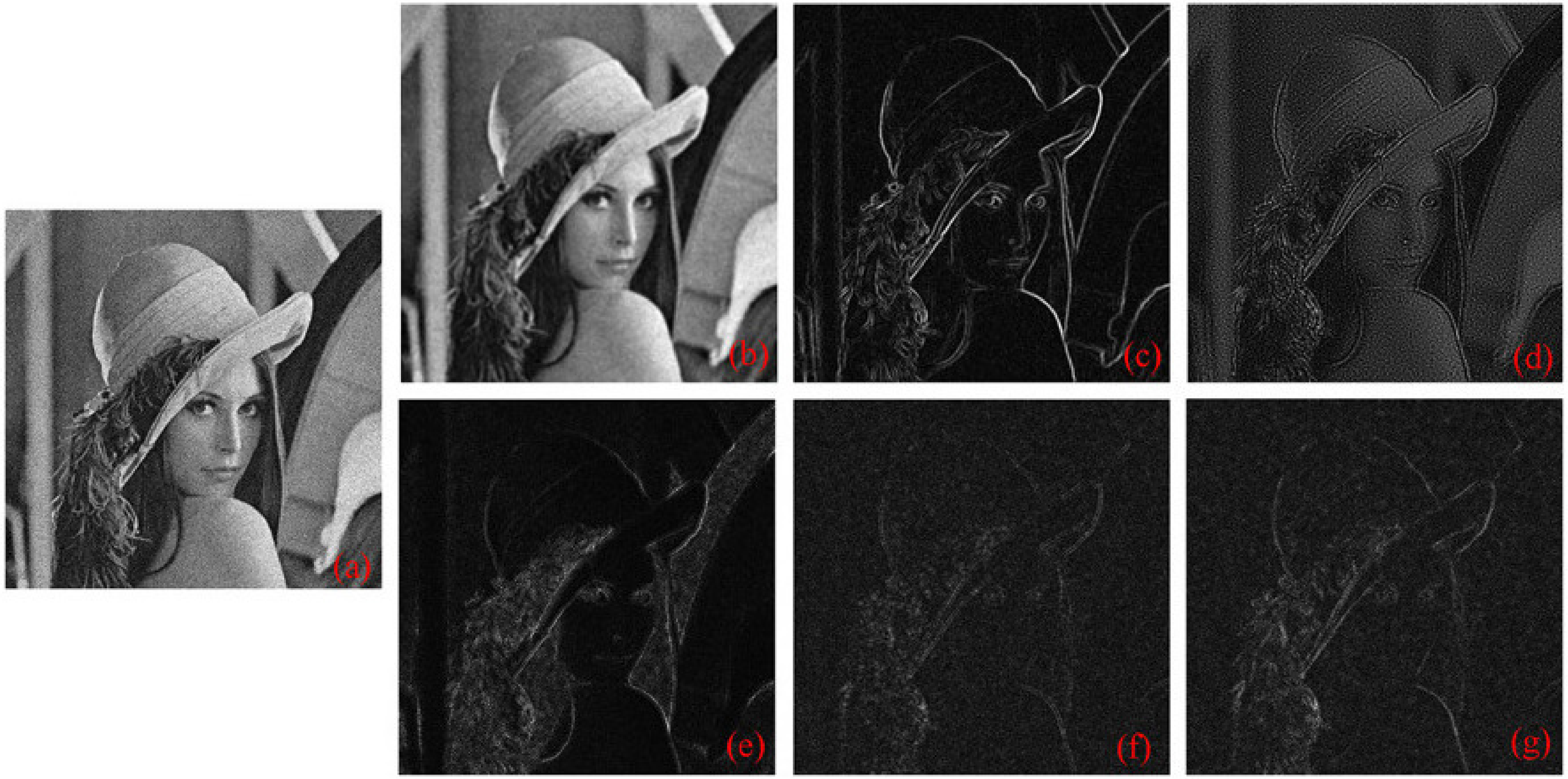
Cartesian moments

$$m_{pq} = \sum_{x=1}^{M} \sum_{y=1}^{N} x^p y^q P_{xy}$$

$$Z_{3,3} = \sqrt{8}\left(3\rho^3 \cos 3\theta\right)$$

http://astro.if.ufrgs.br/telesc/aberracao.htm

Zernike moments



The Lena image with noise ($\sigma = 20$) shown in (a). Radius r = 3.
(b)–(g) are the images of Z_{00}, Z_{11}, Z_{20}, Z_{22}, Z_{31}, Z_{33}.

Moment-based nonlocal filtering

Normalization: $$\hat{Z}_{pq} = \begin{cases} Z_{pq} / Z_{p-2,q} & if\ Z_{p-2,q} \neq 0\ and\ q < p \\ Z_{pq} & if\ Z_{p-2,q} = 0\ or\ q = p \end{cases}$$

Vector for each pixel $v(i) = \left\{ \hat{Z}_1(i), \hat{Z}_2(i), \hat{Z}_3(i), \hat{Z}_4(i), \hat{Z}_5(i), \hat{Z}_6(i) \right\}$

$$\hat{Z}_1 \approx z_{00}; \hat{Z}_2 \approx z_{11}; \hat{Z}_3 \approx z_{20}; \hat{Z}_4 \approx z_{22}; \hat{Z}_5 \approx z_{31}; \hat{Z}_6 \approx z_{33};$$

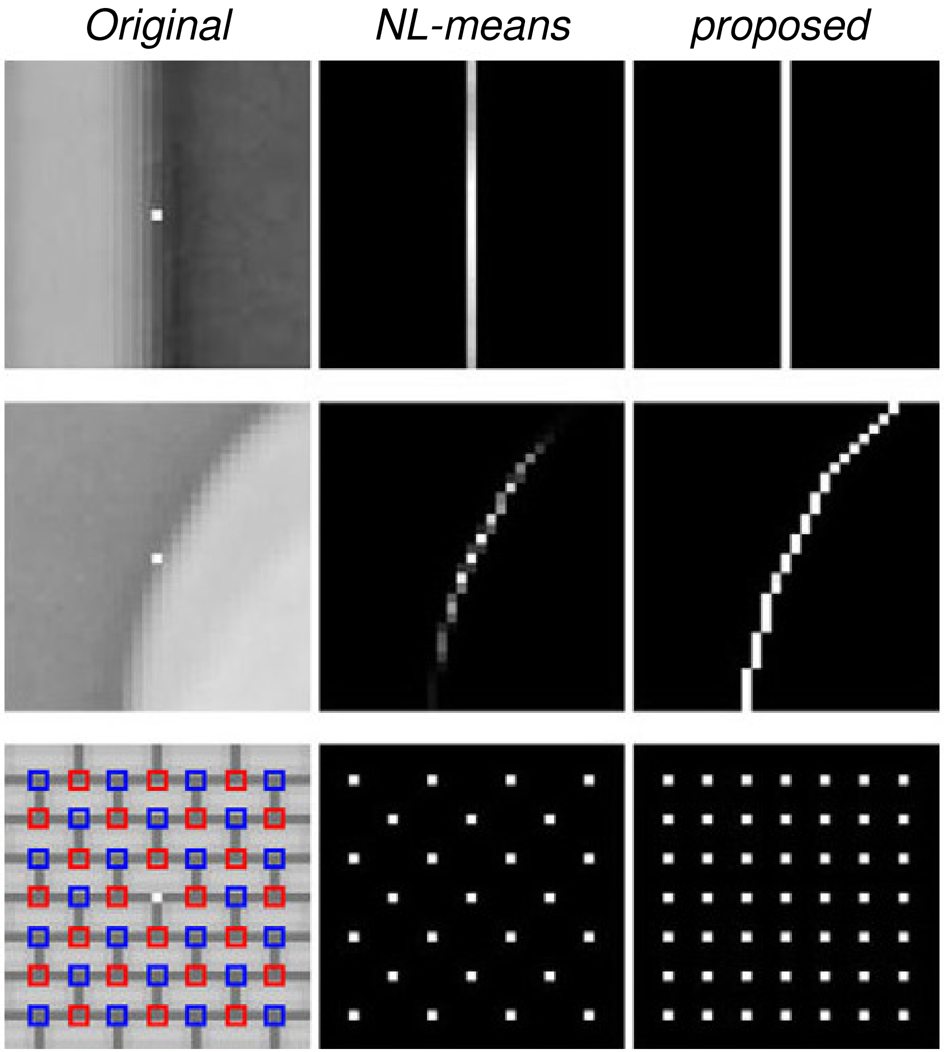
Intensity values: $u(i)$

Similarity measurement: $\sum \left\| v(i) - v(j) \right\|^2$

$$\omega(i, j) = \frac{1}{C(i)} e^{-\frac{\sum \left\| v(i) - v(j) \right\|^2}{h^2}} \qquad C(i) = \sum_j e^{-\frac{\sum \left\| v(i) - v(j) \right\|^2}{h^2}}$$

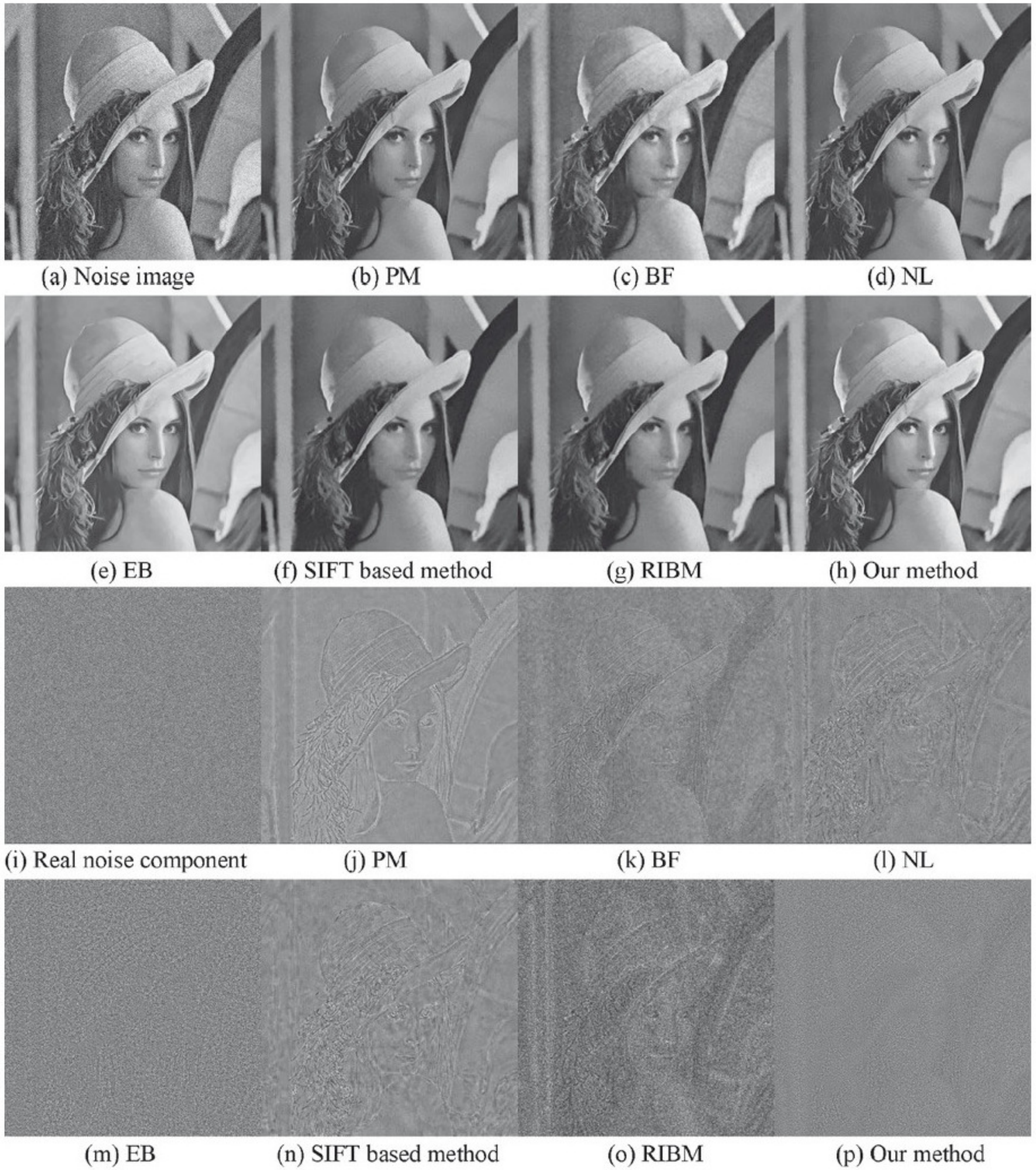
$$NL(u)(i) = \sum_{j \in I} \omega(i, j) u(j) \qquad (h = 95)$$

Weight ($\omega(i,j)$) distribution used to estimate the central pixel



$$\omega(i,j) = \frac{1}{C(i)} e^{-\frac{\left\| u(N_i) - u(N_j) \right\|^2_{G_\rho}}{h^2}}$$

$$\omega(i,j) = \frac{1}{C(i)} e^{-\frac{\sum \left\| v(i) - v(j) \right\|^2}{h^2}}$$



a) Noise σ = 20	(PSNR = 22.16)
b) PM model	(28.83)
c) Bilateral f.	(29.16)
e) NL-means	(31.09)
e) Exemplar-based method	(**32.64**)
f) SIFT based m.	(31.26)
g) rotationally invariant block matching	(31.75)
h) Our	(**32.29**)
(block matching and 3D f.	*(**33.05**))*

i) real noise component

j) – p) corresponding noise component of each method

$$PSNR = 10\log_{10} \frac{255^2}{\sum_{i \in I} \left(NL(u)(i) - u_0(i) \right)^2 / |I|} [dB]$$

PSNR [dB]

Method	Lena	Barbara	Boat	House	Peppers
Our method	32.29	30.14	**30.20**	**33.12**	**30.98**
PM model [1]	28.83	25.38	27.36	29.88	29.28
BF [3]	29.16	27.02	28.41	29.76	29.42
NL-means [4]	31.09	29.38	28.60	31.54	29.05
EB method [12]	**32.64**	**30.27**	30.17	33.03	30.80
SIFT based [6]	31.26	29.81	29.11	32.09	29.28
RIBM [9]	31.75	29.87	29.39	32.26	29.88
BM3D [13]	***33.05***	***31.78***	***30.88***	***33.77***	***31.29***





Questions?