

# Handling Blur

## Image Restoration

Jan Flusser, [Filip Sroubek](#), Barbara Zitova

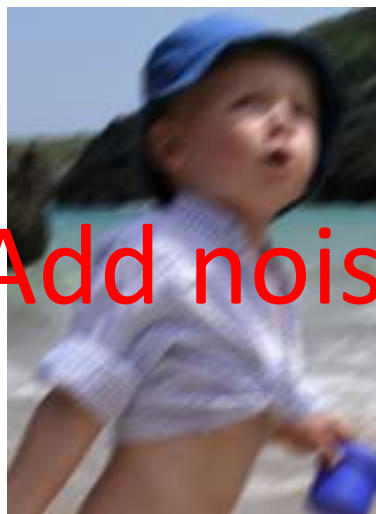
Institute of Information Theory and Automation  
Academy of Sciences of the Czech Republic  
Prague

# Contents

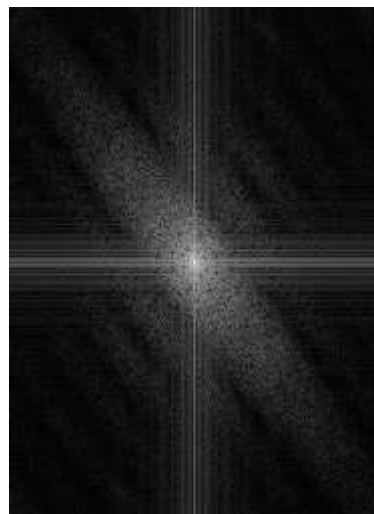
- Non-blind deconvolution
  - Wiener, Energy minimization, regularization
- Blind deconvolution
  - MAP, VB
- Multichannel (multiframe)
  - Super-resolution
- Space-variant blind deconvolution
  - Patch-wise, Parametric, Object motion, Conversion to space-invariant
- Hybrid methods
  - Fusion approach, High-speed cameras, Inertial sensors

# Inverse Filter

Add noise



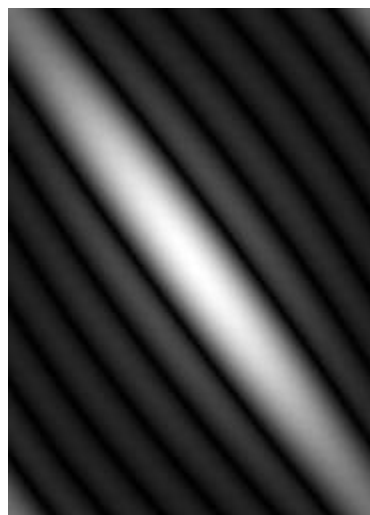
$z(x)$



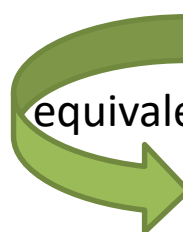
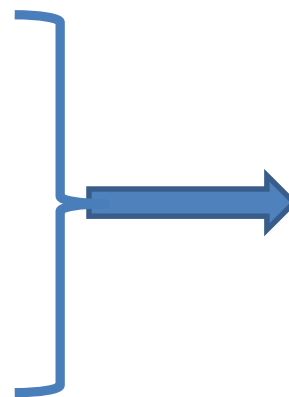
$Z(\omega)$



$h(x)$



$H(\omega)$



equivalent

$$\tilde{U}(\omega) = \frac{Z(\omega)}{H(\omega)}$$

$$\tilde{u} = \arg \min_u \frac{1}{2} \|h * u - z\|^2$$

# Wiener Filter



$z(x)$

Replace by a constant

$$\tilde{U} = \frac{H^*}{|H|^2 + \frac{S_v}{S_u}} Z$$

equivalent

$$\tilde{u} = \arg \min_u \frac{1}{2} \|h * u - z\|^2 + \lambda \|u\|^2$$



$\lambda = 0.001$



$\lambda = 0.1$

# Deconvolution as energy minimization

$$\min_u E(u) = \min_u \frac{1}{2} \|u * h - z\|^2 + \lambda Q(u)$$

Data term                      Regularization

- Regularization: ill-posed problem --> well posed

Enforce image smoothness

– Wiener:  $Q(u) = \int u(x)^2 dx$

– Tikhonov:  $\int |\nabla u(x)|^2$

– Total Variation:  $\int |\nabla u(x)|$

– Non-convex  $L_p$  quasi-norm:  $\int |\nabla u(x)|^p, \quad p < 1$

# Regularization

Different regularization  $Q(u)$

$$|\nabla u|^2$$

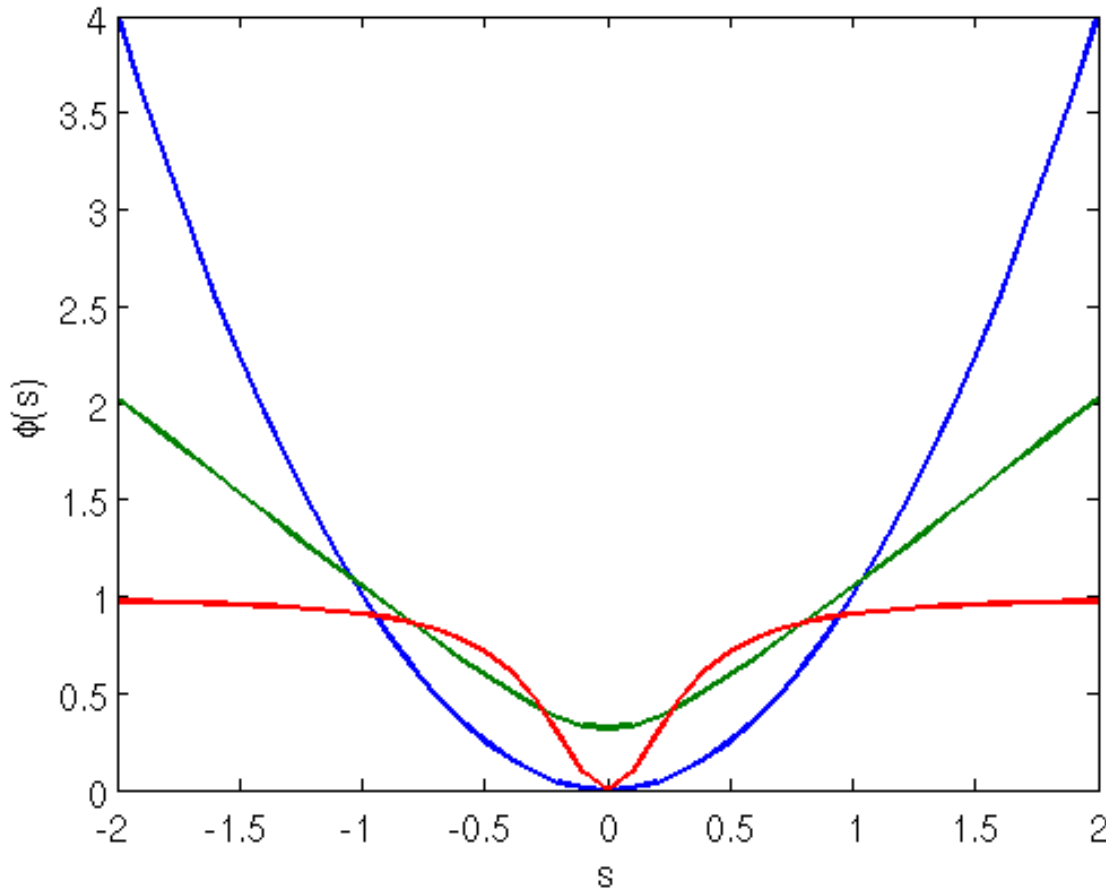
$$|\nabla u|^1$$

$$|\nabla u|^0$$



Estimated image  $\tilde{u}(x)$

# Regularization

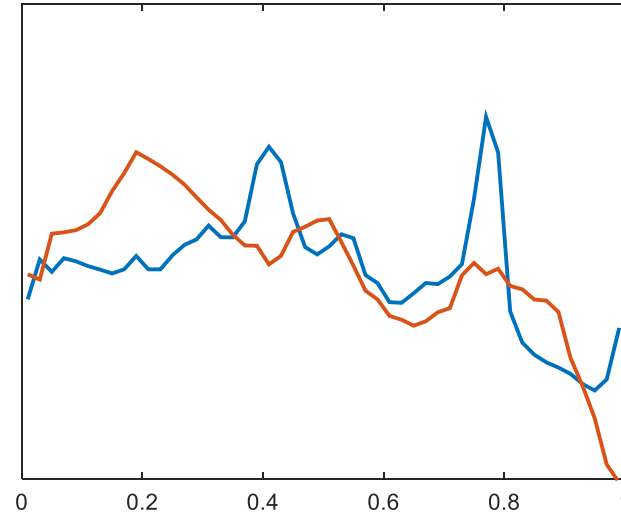


Tikhonov ...  $|s|^2$

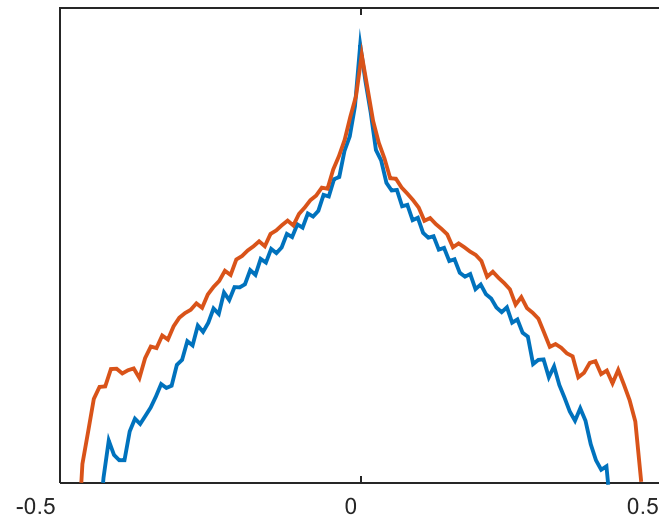
TV ...  $|s| \approx \sqrt{|s|^2 + \epsilon}$

$l_0$ -norm ...  $|s|_0 \approx \frac{|s|^2}{|s|^2 + \epsilon}$

# Statistics of sharp images



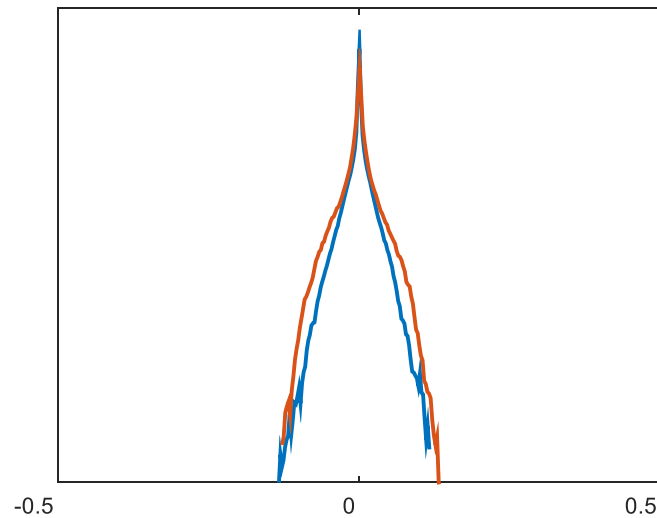
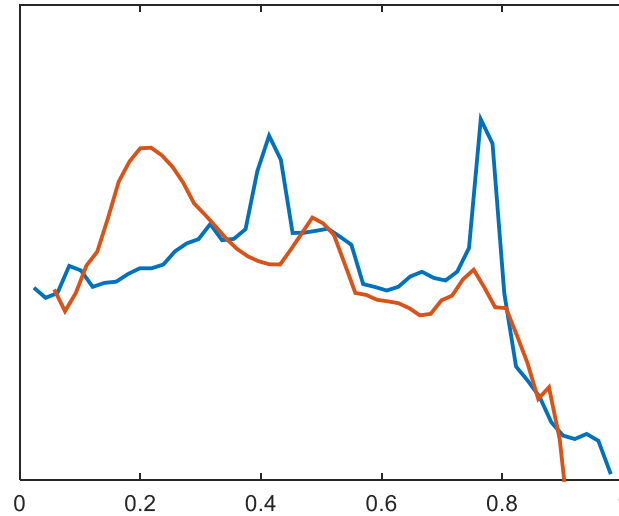
Intensity



Gradient



# Statistics of blurred images



# How to tackle the blind case?

- When the blur kernel  $h(x)$  is not known.
- One is tempted to:
  - 1) Add blur regularization
  - 2) Perform alternating minimization



# Alternating Minimization

$$\min_{u,h} E(u, h) = \min_{u,h} \frac{1}{2} \|h * u - z\|^2 + \lambda Q(u) + \gamma R(h)$$

- Alternating Minimization

1. *u-step*:  $\tilde{u} = \arg \min_u E(u, \tilde{h})$

2. *h-step*:  $\tilde{h} = \arg \min_h E(\tilde{u}, h)$

3. *repeat 1 and 2.*

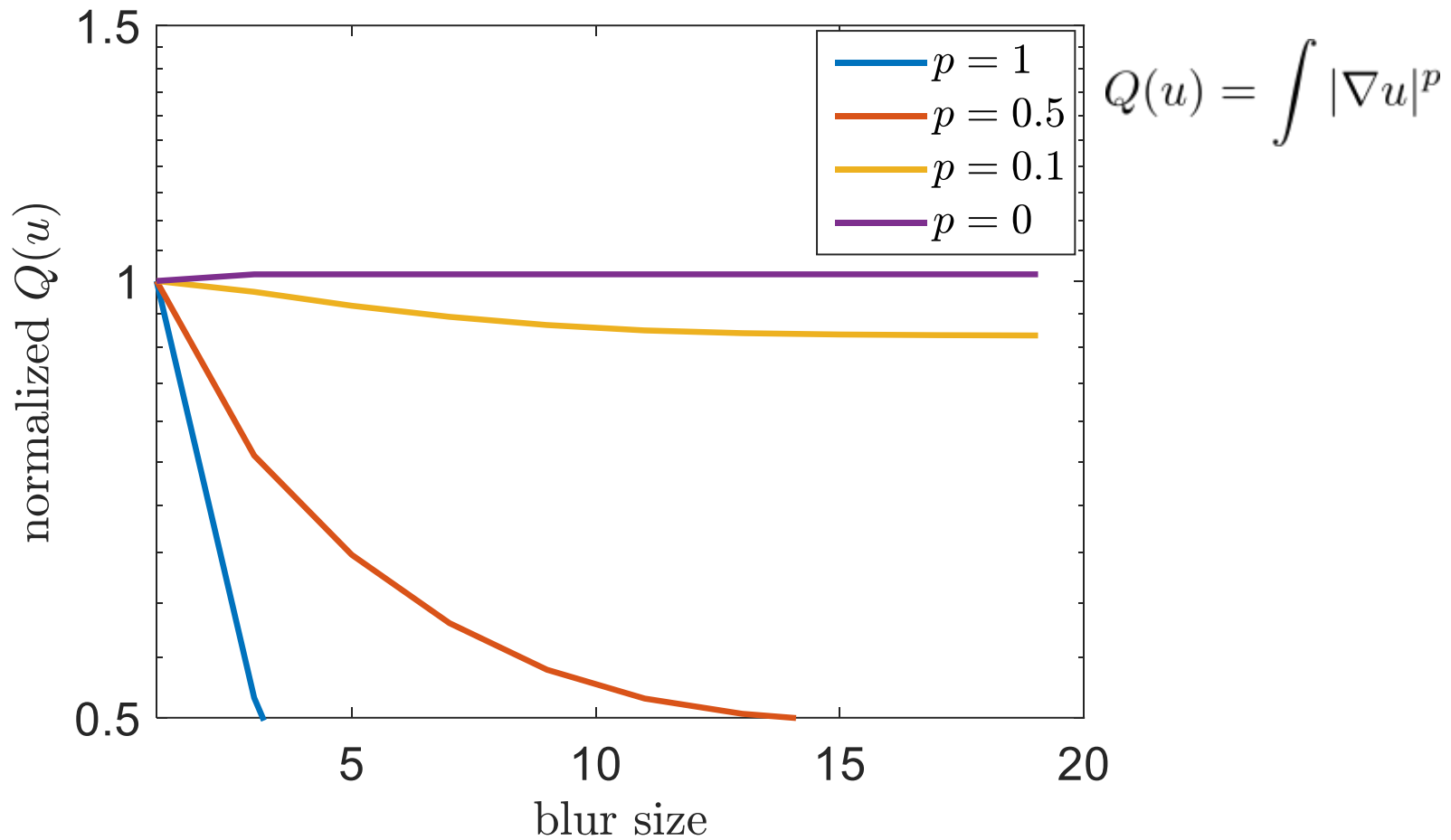
# No-blur solution

$$\min_{u,h} \frac{1}{2} \|u * h - z\|^2$$

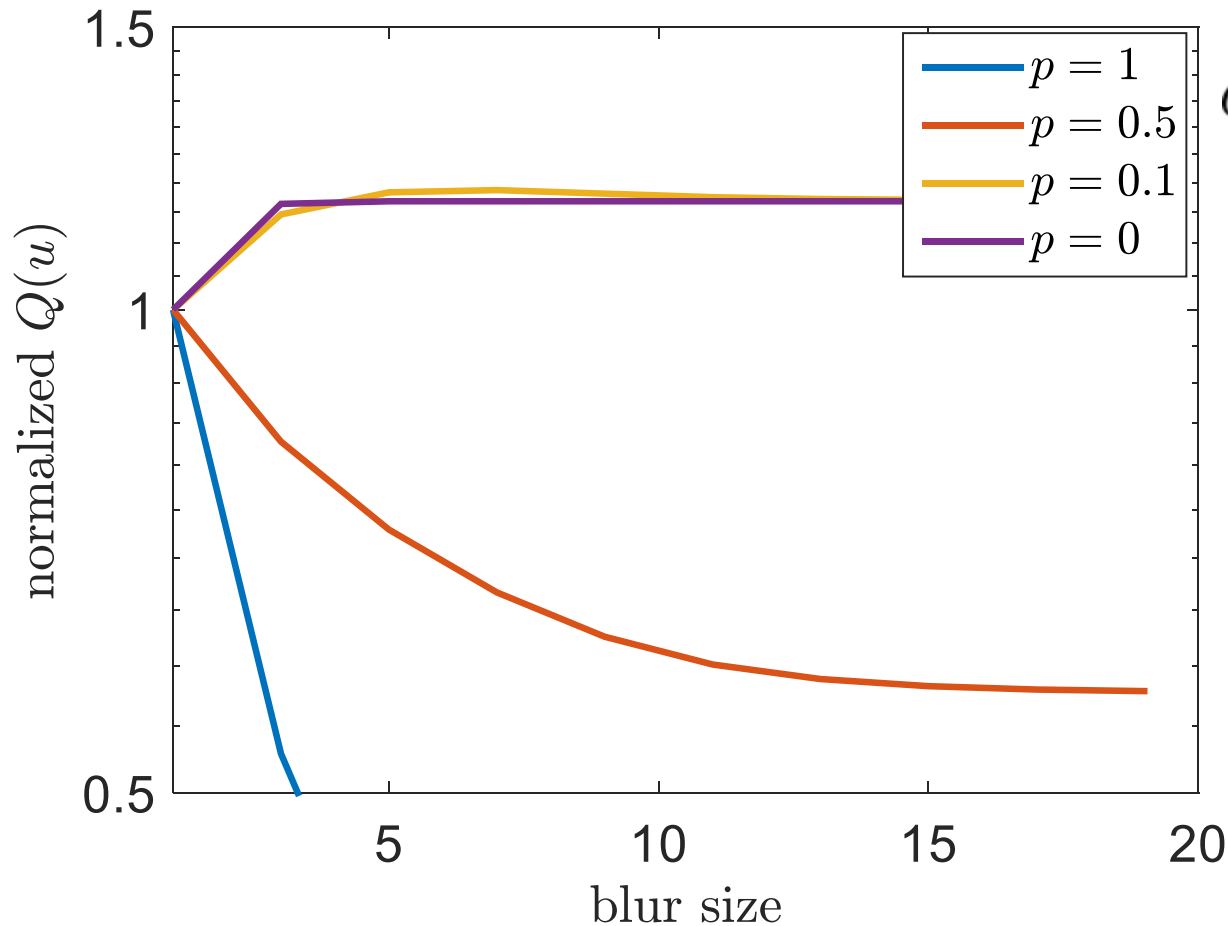
- In theory any pair  $(\tilde{u}, \tilde{h})$  is a solution  $\rightarrow$  remember inverse filter
- common regularization  $\rightarrow$  “**NO-BLUR**” solution

$$\tilde{u}(x) = z(x), \quad \tilde{h}(x) = \delta(x)$$

# Regularization favors blur



# Regularization favors blur



$$Q(u) = \int |\nabla u|^p$$

Artificially  
sparsify  
images

# We need tricks!

- To avoid “no-blur” solution:
  - Artificial sharpening
  - Remove spikes
  - Adjusting priors on the fly
  - Hierarchical approach

Chan TIP1998  
Shan SigGraph08  
Cho SigGraph 09  
Xu ECCV09, 13  
Almeida TIP10  
Krishnan 11  
Zhong 13  
Sun 13  
Michael 14  
Perrone 15  
Pan 16

# Artificial Sharpening

Blurred image



Shock filter



Blur prediction  
h-step



Deconvolution  
u-step



Cho et al., SIGGRAPH 2009

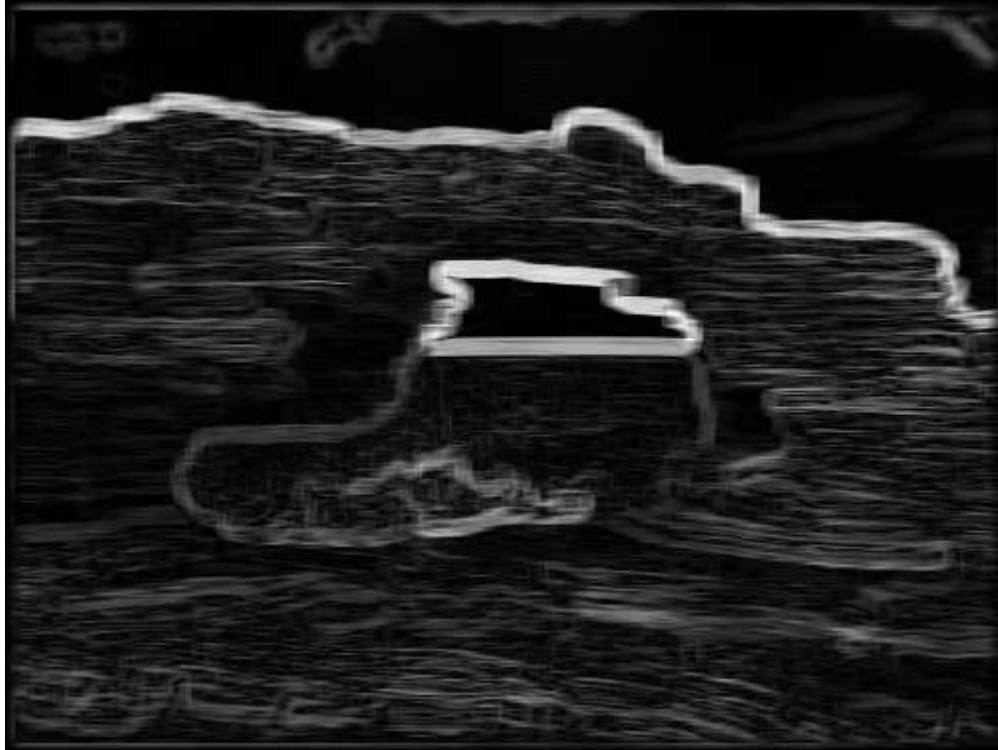


# Remove Spiky Objects



Xu et al., ECCV 2010

# Remove Spiky Objects



Mask out small objects

Xu et al., ECCV 2010

# Remove Spiky Objects



Reconstructed image with  
small objects removed

Xu et al., ECCV 2010

# Adjusting priors



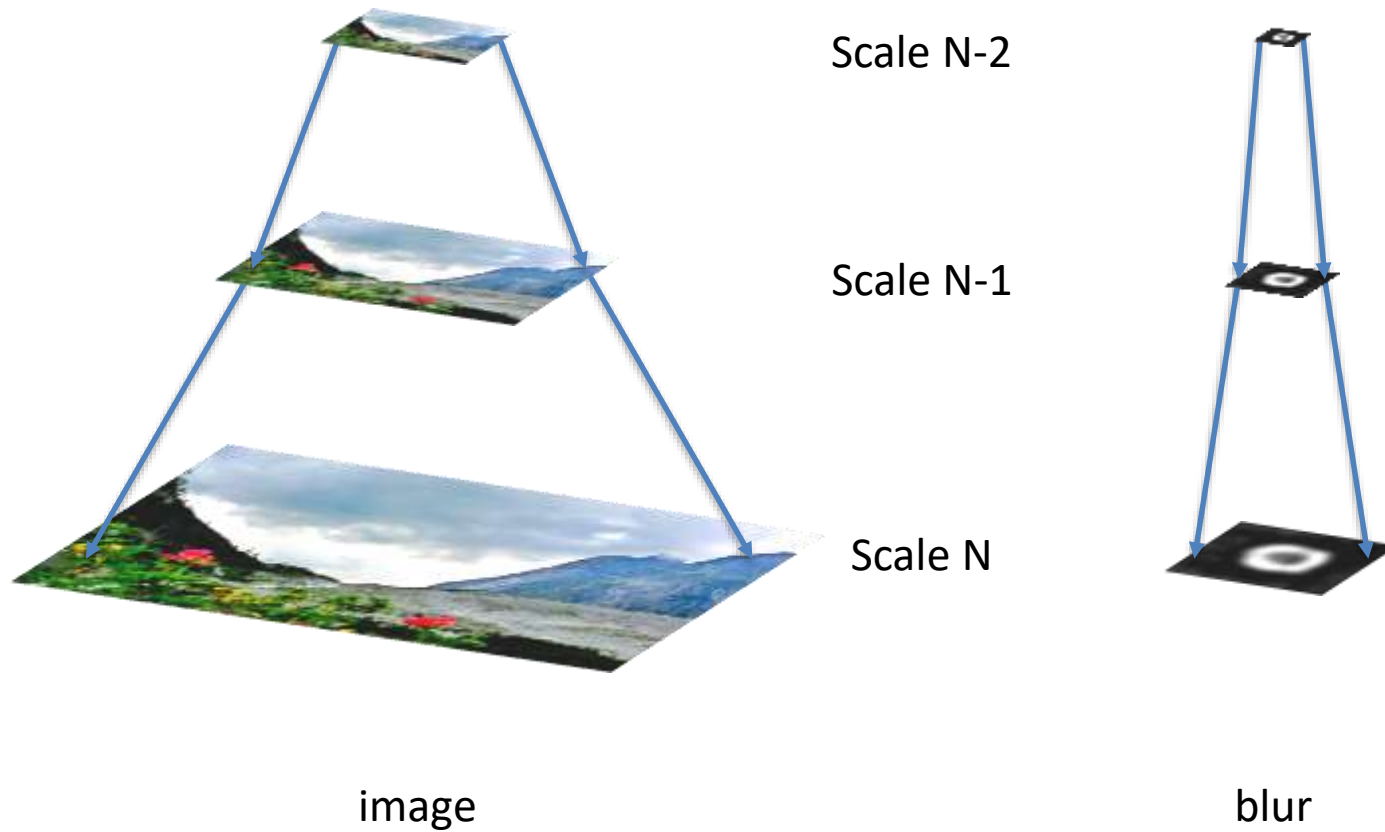
$$u \quad Q(u) = \sum_i |\nabla u_i|^2$$

$$\sum_i |\nabla u_i|^1$$

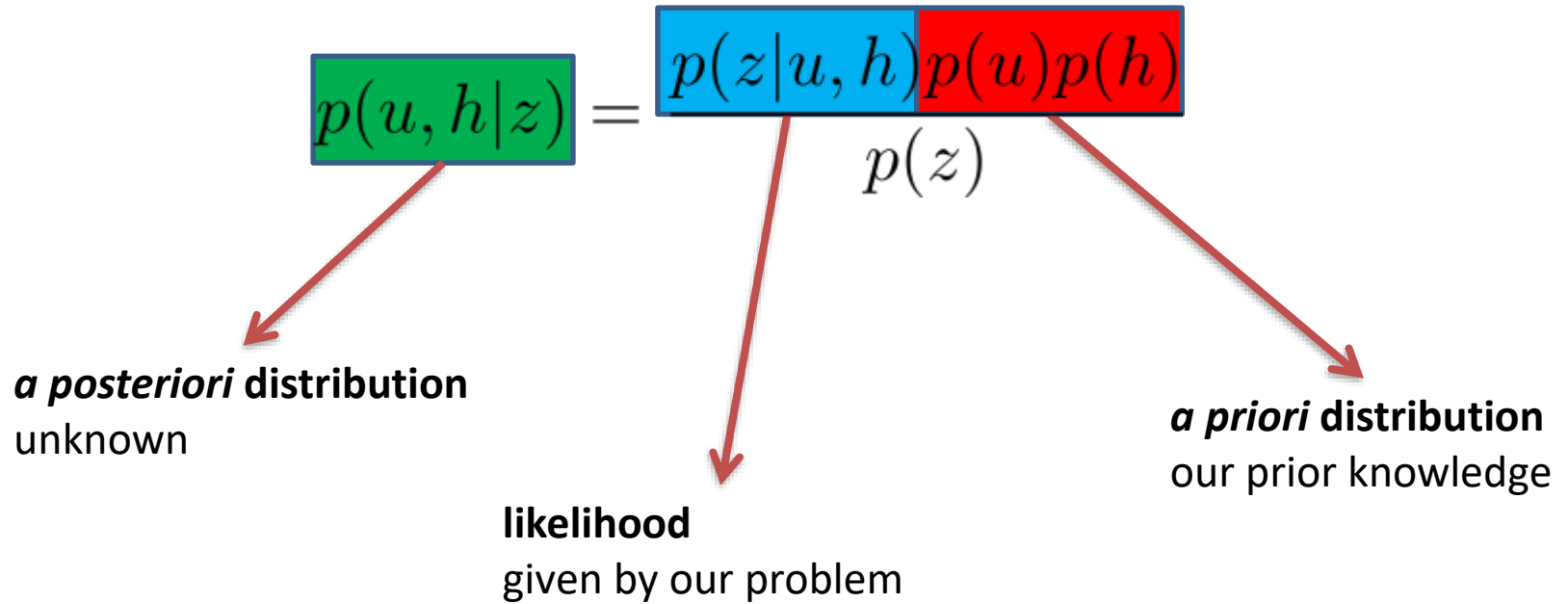
$$\sum_i |\nabla u_i|^{0.5}$$

- Start with an overestimated noise level and slowly decrease it to the correct level.
- Start with  $p \ll 1$  and slowly increase it to  $p=1$ .

# Hierarchical Deconvolution



# Bayesian Paradigm



- Maximum a posteriori (MAP):  $\max p(u, h|z)$

# Blind deconvolution with MAP

- max *a posteriori* probability  $p(u, h|z)$

$$\implies \min -\log p(u, h|z)$$

$$-\log p(u, h|z) \propto \boxed{-\log p(z|u, h)} \boxed{-\log p(u) - \log p(h)}$$

- Exponential family

$$E(u, h) = \frac{\lambda}{2} \|u * h - z\|^2 + Q(u) + R(h)$$

**NOTHING NEW!**

# Bayesian Paradigm revisited

- Marginalize the posterior

$$p(h|z) = \int p(u, h|z) du$$

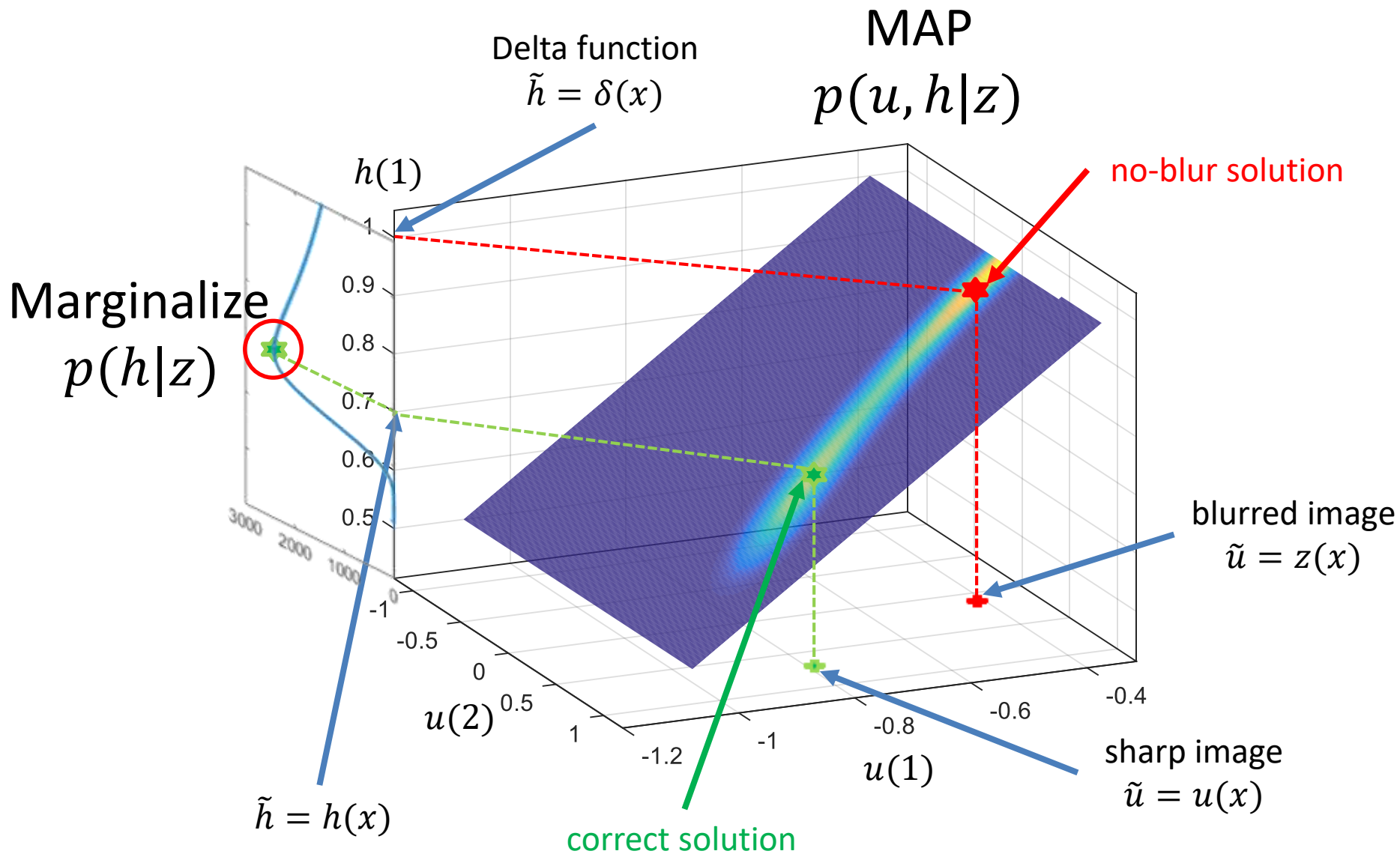
- Maximize the marginalized prob.

$$\hat{h} = \arg \max_h p(h|z)$$

- and then maximize the posterior

$$\hat{u} = \arg \max_u p(u, \hat{h}|z)$$





# How to marginalize?

$$p(h|z) = \int p(u, h|z) du$$

- If Gaussian distributions  $\rightarrow$  analytic solution exists in the form of Gaussian distribution
- If not (our case)  $\rightarrow$  approximation
  - Laplace approximation
  - Factorization with Variational Bayes

# Variational Bayes

- Factorization of the posterior

$$p(u, h|z) \approx q(u)q(h)$$

and then marginalization is trivial.

Miskin 01  
Fergus 06  
Whyte 10  
Levin 11  
Babacan 12  
Wipf 14

- Every factor  $q$  depends on moments of other variables => must be solved iteratively.

# Example of blind deconvolution

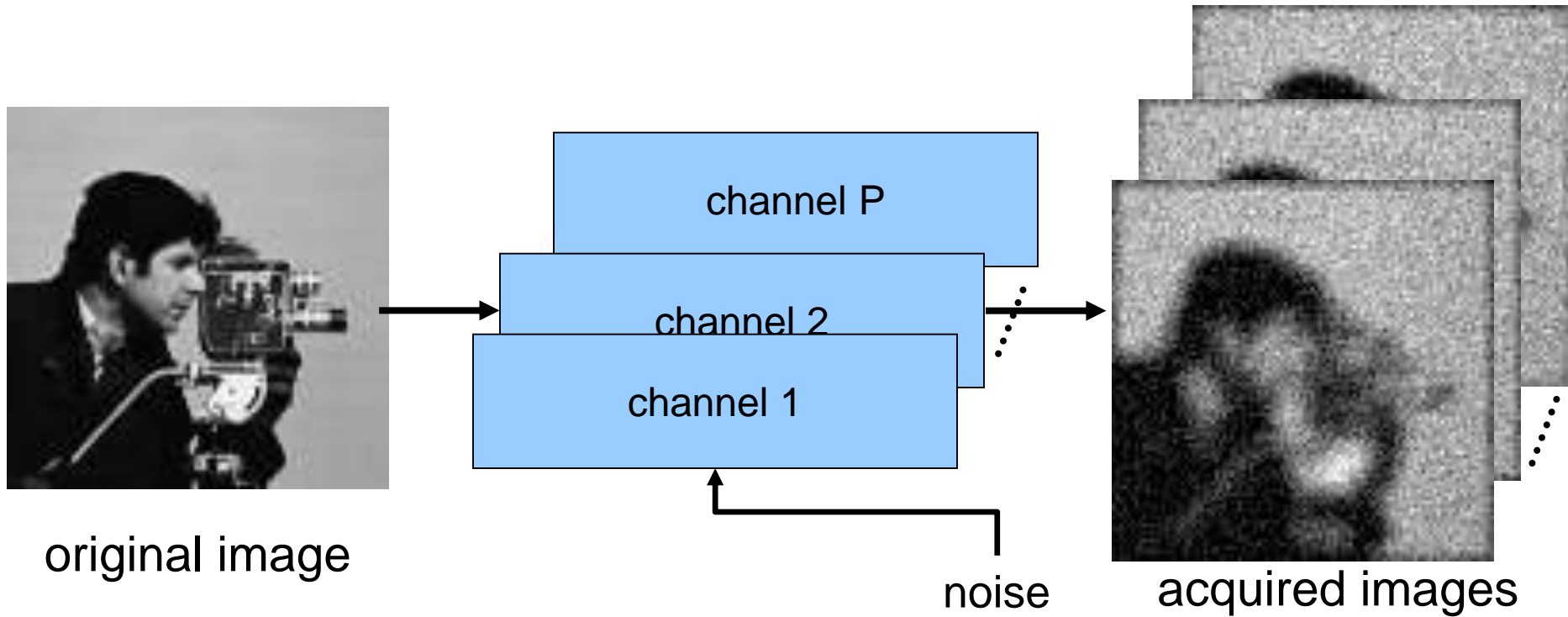


Blurred image  
 $z(x)$



Reconstructed image  
 $\tilde{u}(x)$

# Multichannel Model



$$[u * h_p] + n_p = z_p$$

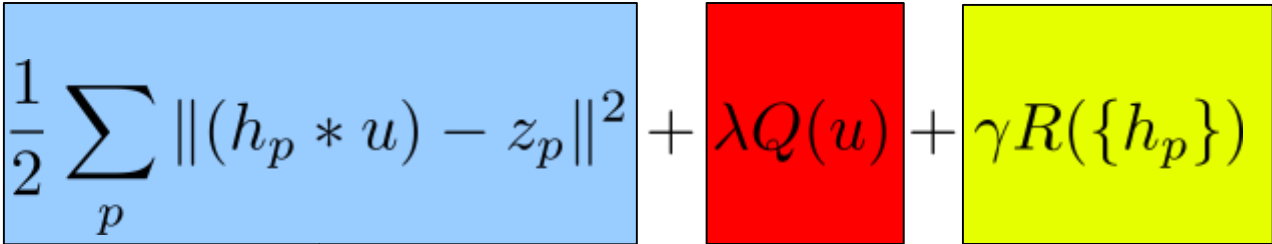
# Image Restoration

- Acquisition model:

$$\begin{aligned} z_1 &= (h_1 * u) + n_1 \\ &\vdots \\ z_P &= (h_P * u) + n_P \end{aligned}$$

- Optimization problem

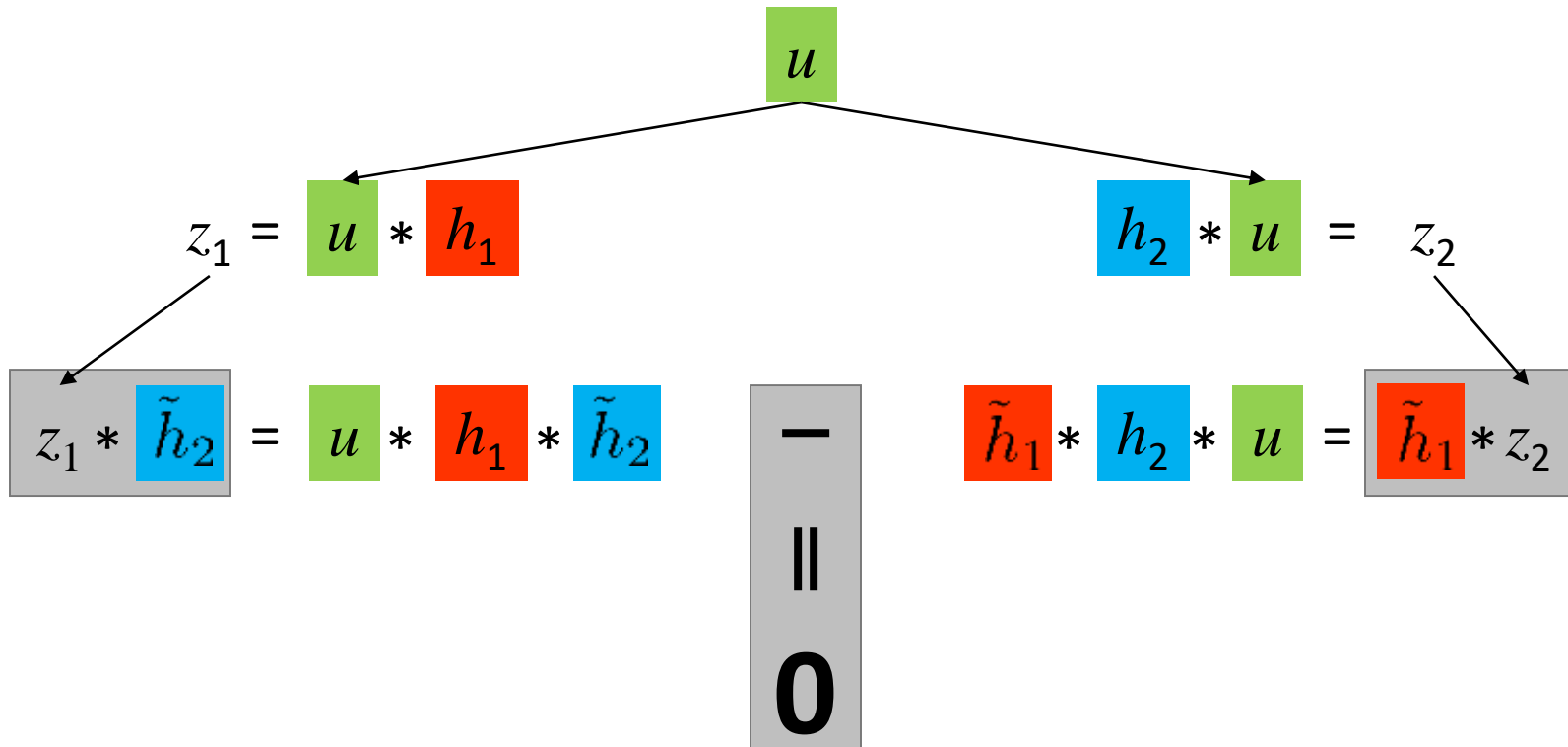
$$E(u, \{h_p\}) = \frac{1}{2} \sum_p \|(h_p * u) - z_p\|^2 + \lambda Q(u) + \gamma R(\{h_p\})$$



Data term                      Image regularization term                      Blur Regularization term

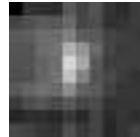
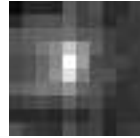
Gaussian noise L2 norm                       $Q(u) = \int \phi(|\nabla u(x)|)$

# Multichannel Blur Regularization



$$R(\{h_p\}) = \frac{1}{2} \sum_{1 \leq p, q \leq P} \|z_p * h_q - z_q * h_p\|^2$$

# Out-of-focus Camera



reconstructed



in focus

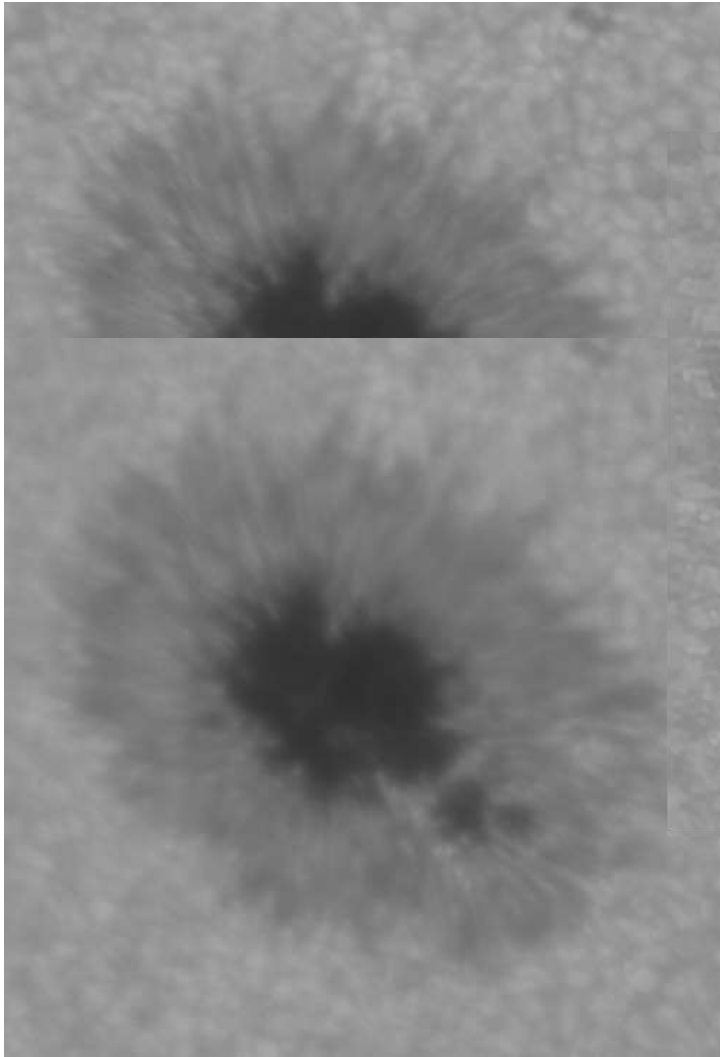


# Camera motion

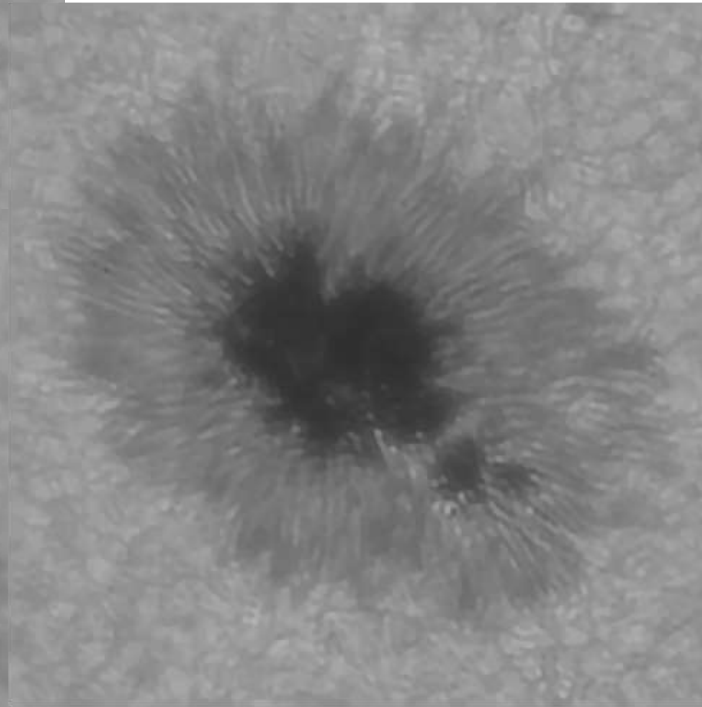


# Astronomical images

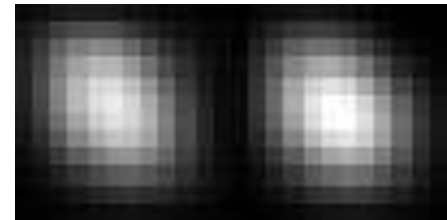
Degraded images



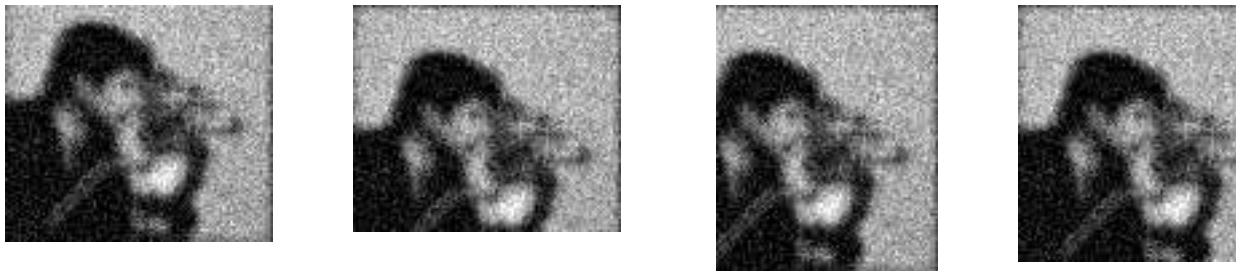
Reconstructed image



PSF estimation



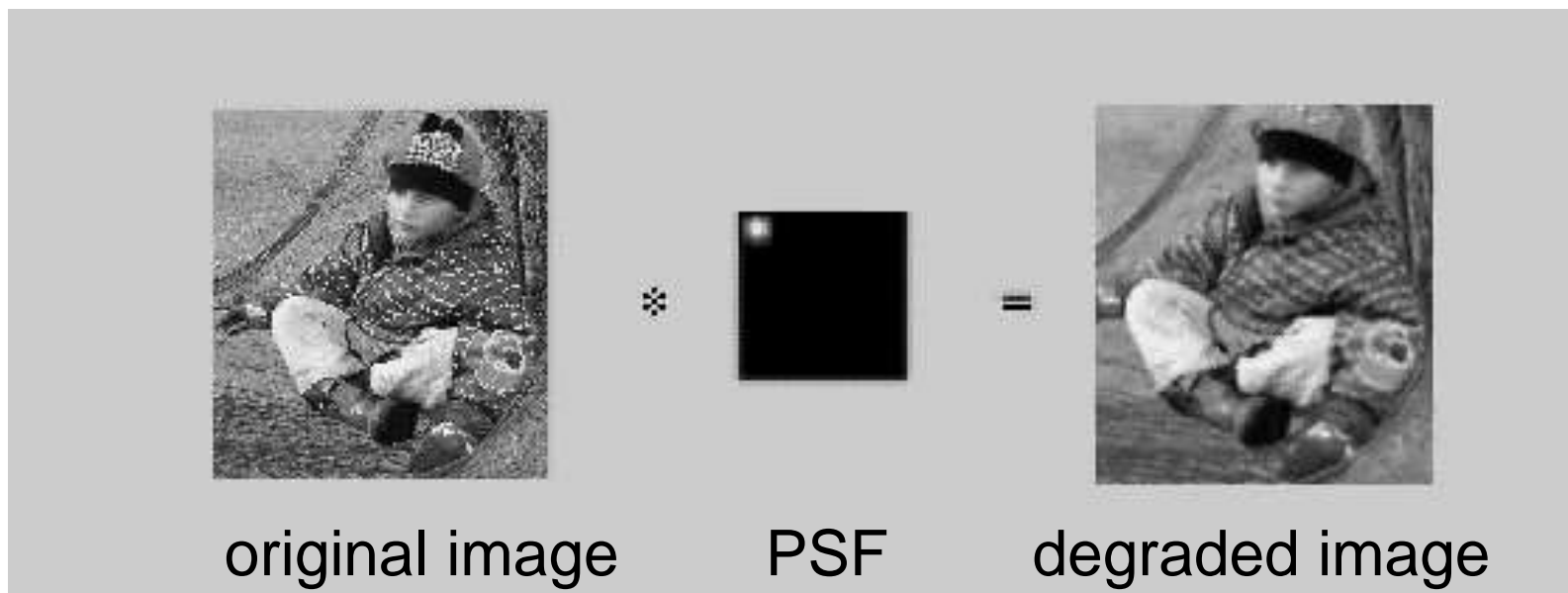
# Misregistration of the channels



... leads to artefacts if not handled properly



# Robustness to misalignment

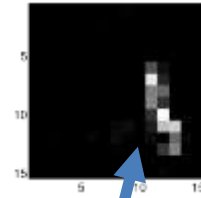


$$(u * h_p)[\tau_p(x)] + n_p(x) = z_p(x)$$

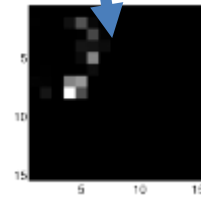
$$(u * \hat{h}_p)(x) + n_p(x) = z_p(x)$$



Slight misalignment



Misalignment compensated



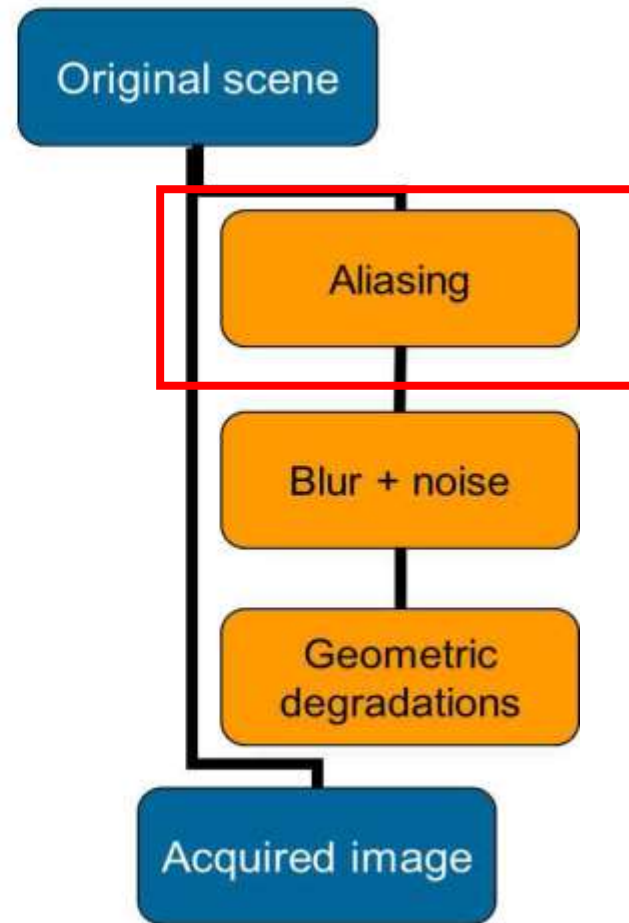


No compensation



With compensation

# Image acquisition with aliasing



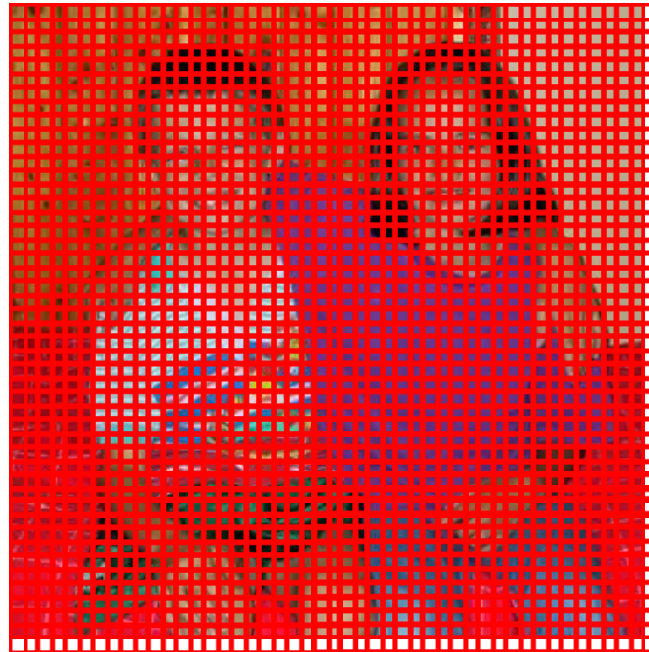


# Aliasing

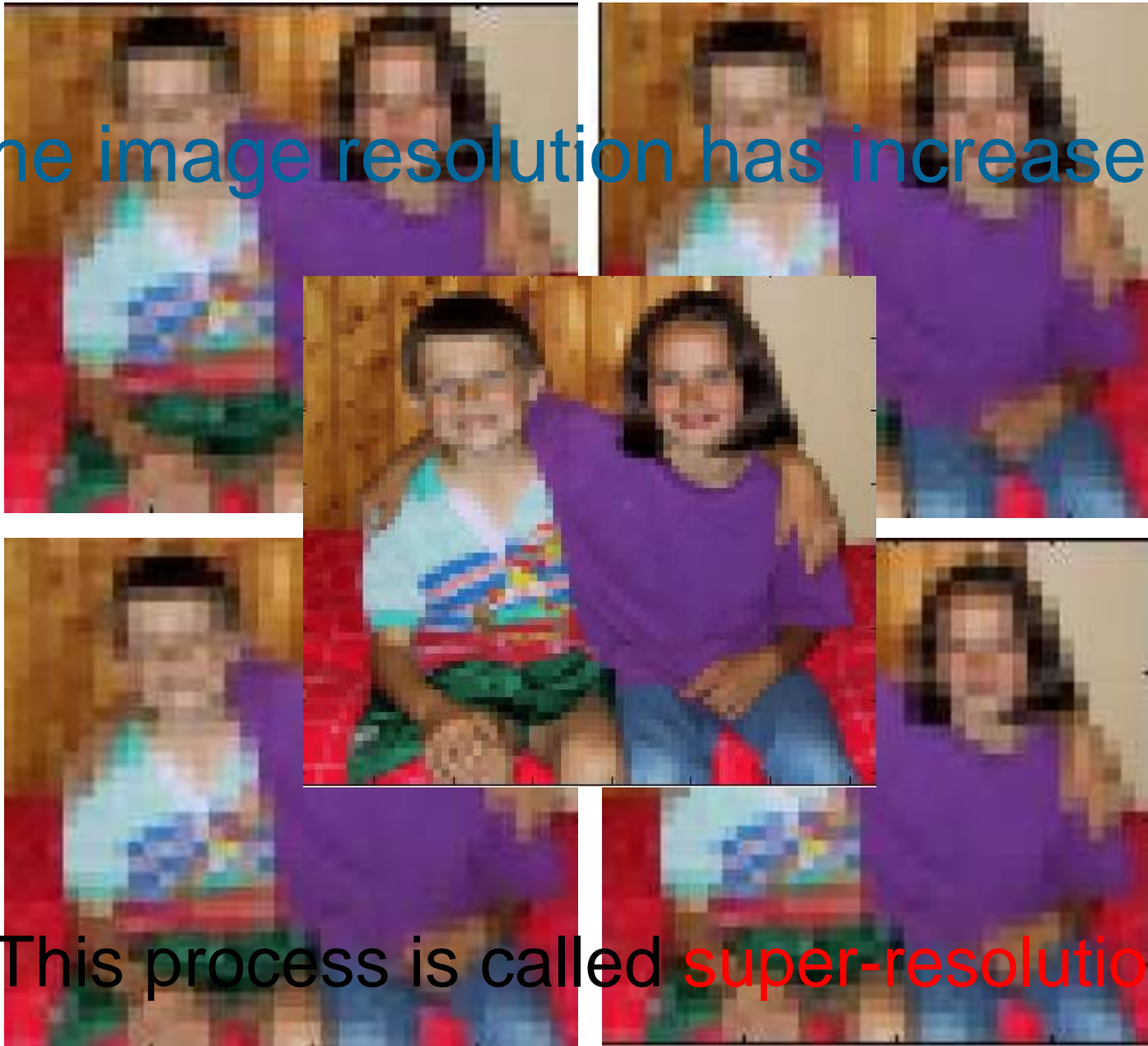
- The loss of the high frequencies (details) due to an insufficient resolution of the camera



To suppress aliasing, apply multiple acquisitions with sub-pixel shifts



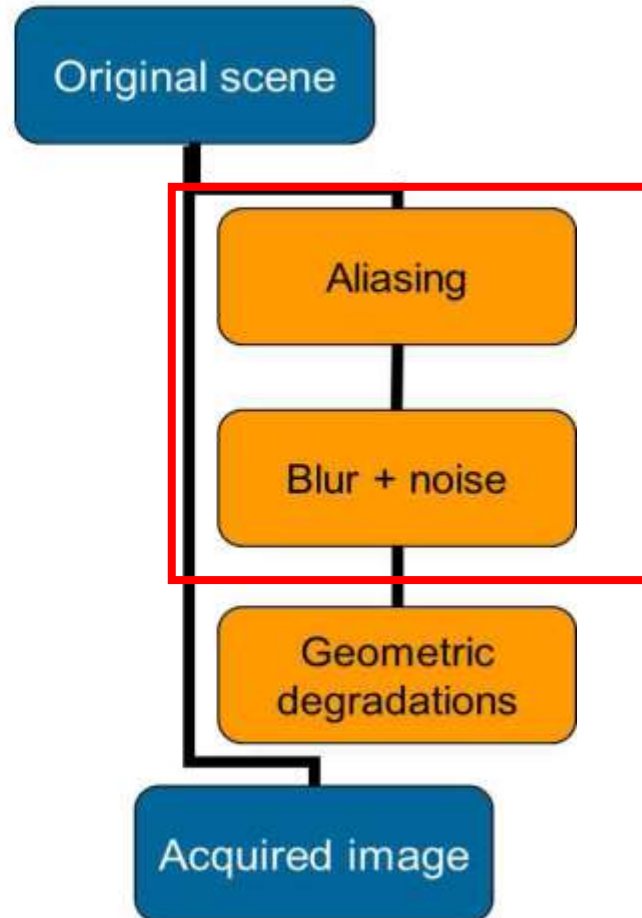
The image resolution has increased



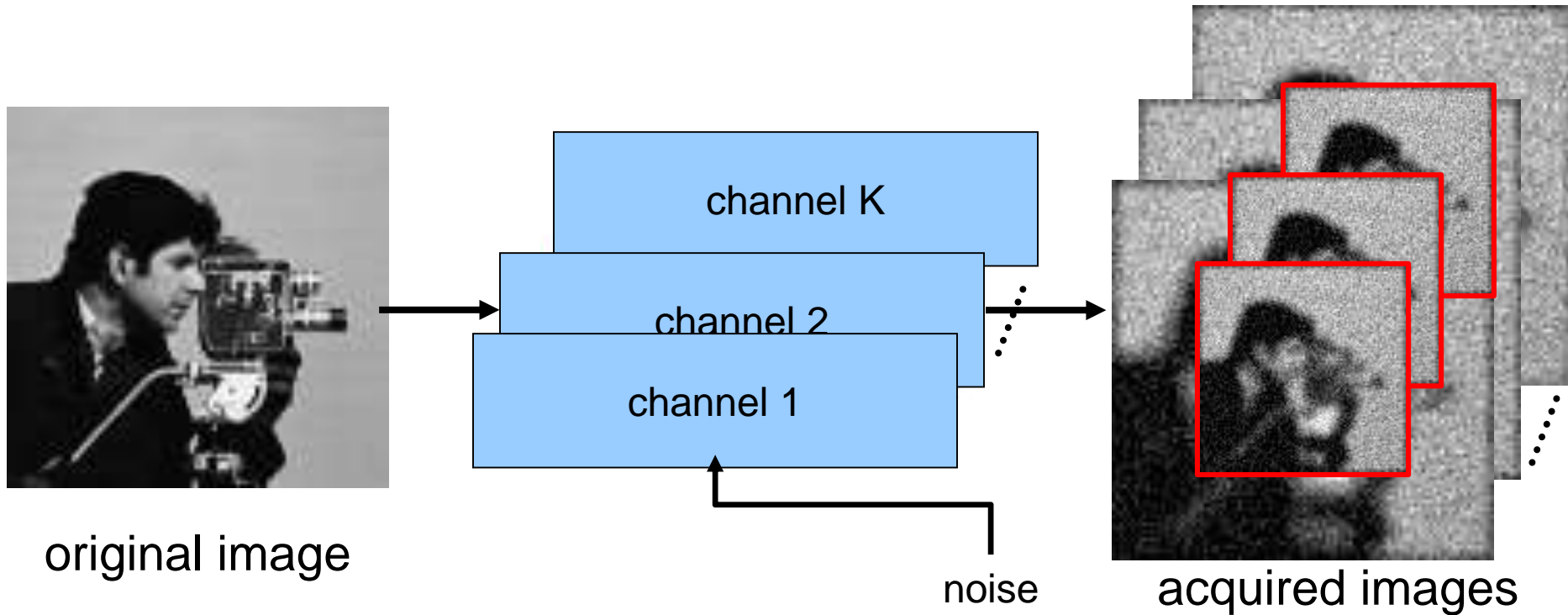
This process is called **super-resolution**

# Realistic superresolution

SR must  
include also  
de-blurring



# MC Model with Decimation



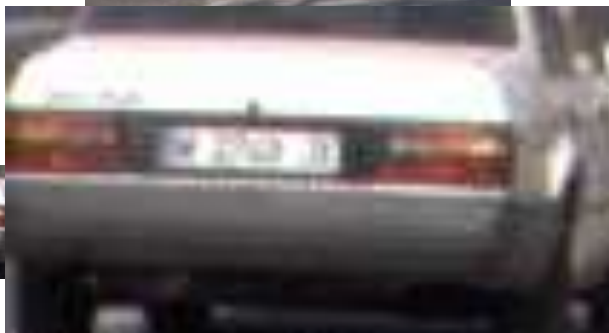
$$D[u * h_k] + n_k = z_k$$

$$\min_{u, h} E(u, h) = \min_{u, h} \frac{1}{2} \sum_k \|D(h_k * u) - z_k\|^2 + \lambda Q(u) + \gamma R(h)$$

# Superresolution



rough registration



Superresolved image (2x)

Optical zoom (ground truth)

8 images



original



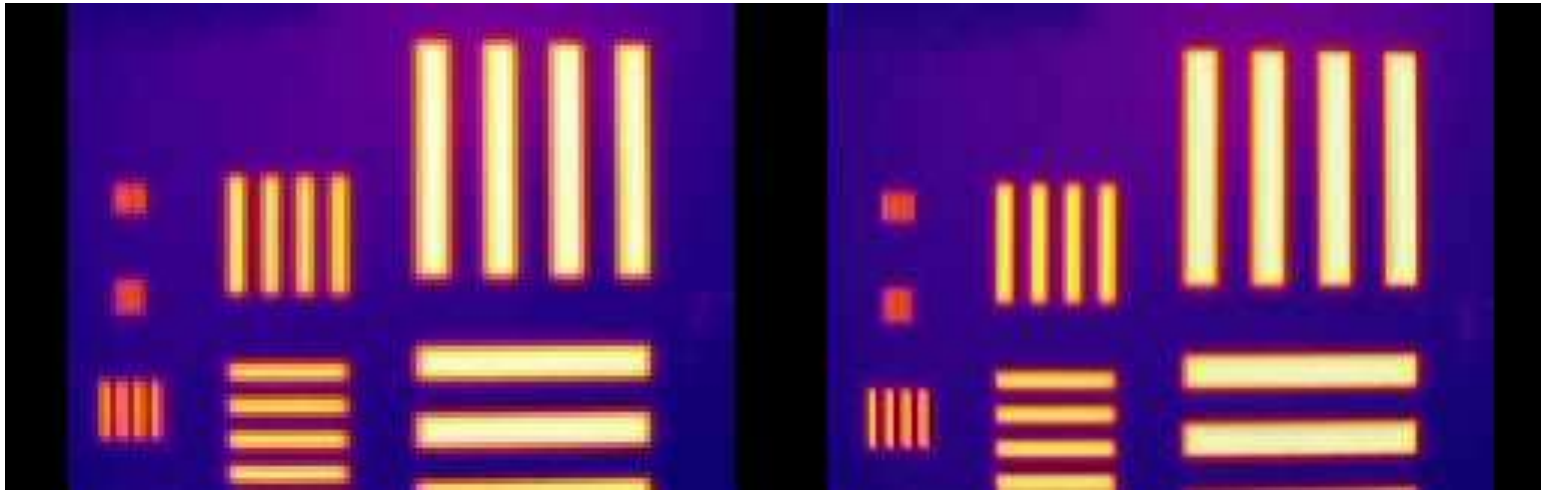
SR 2x



SR 3x



# Video Restoration



Input video

Super-resolution



# Space-variant Blurs



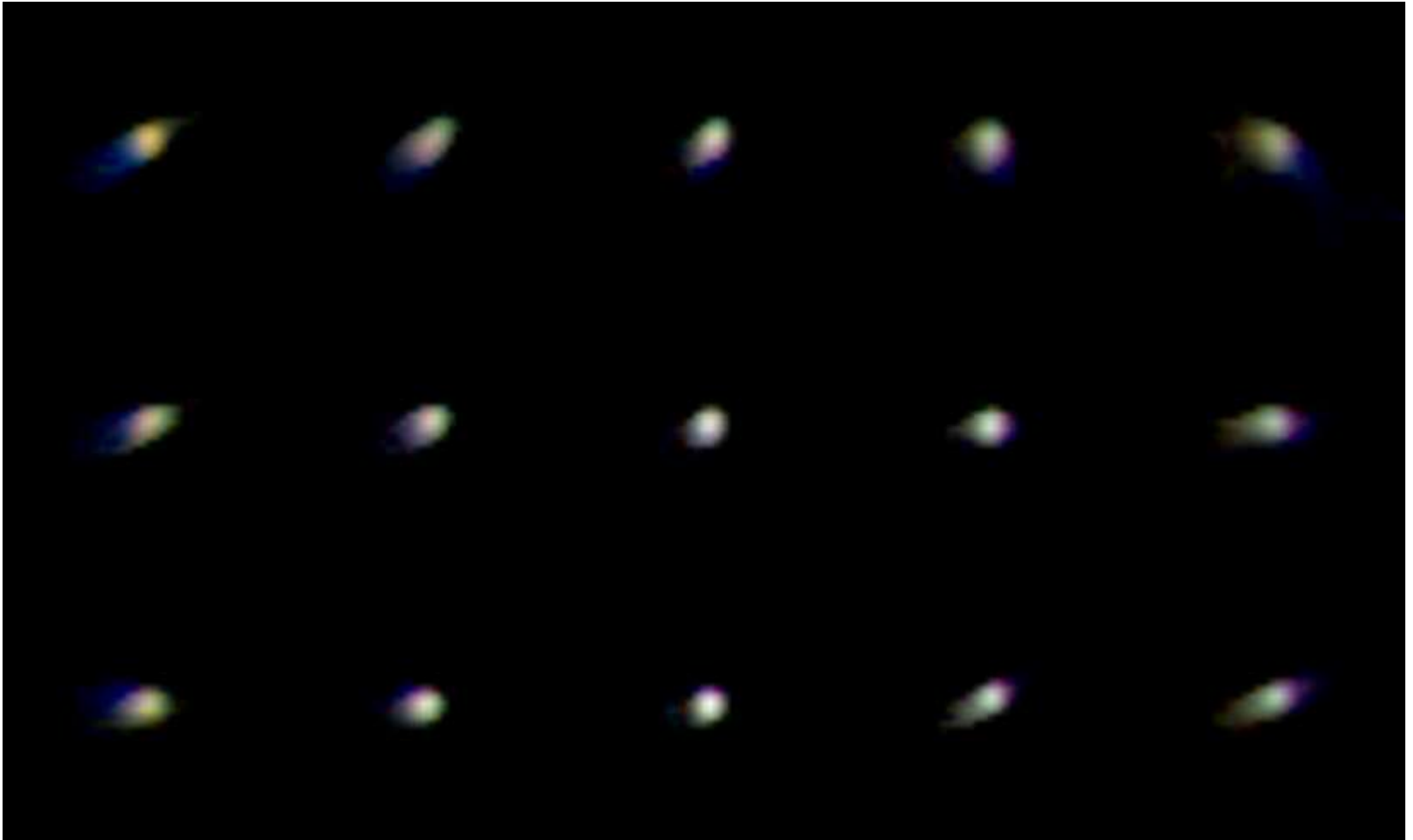
# Camera Motion



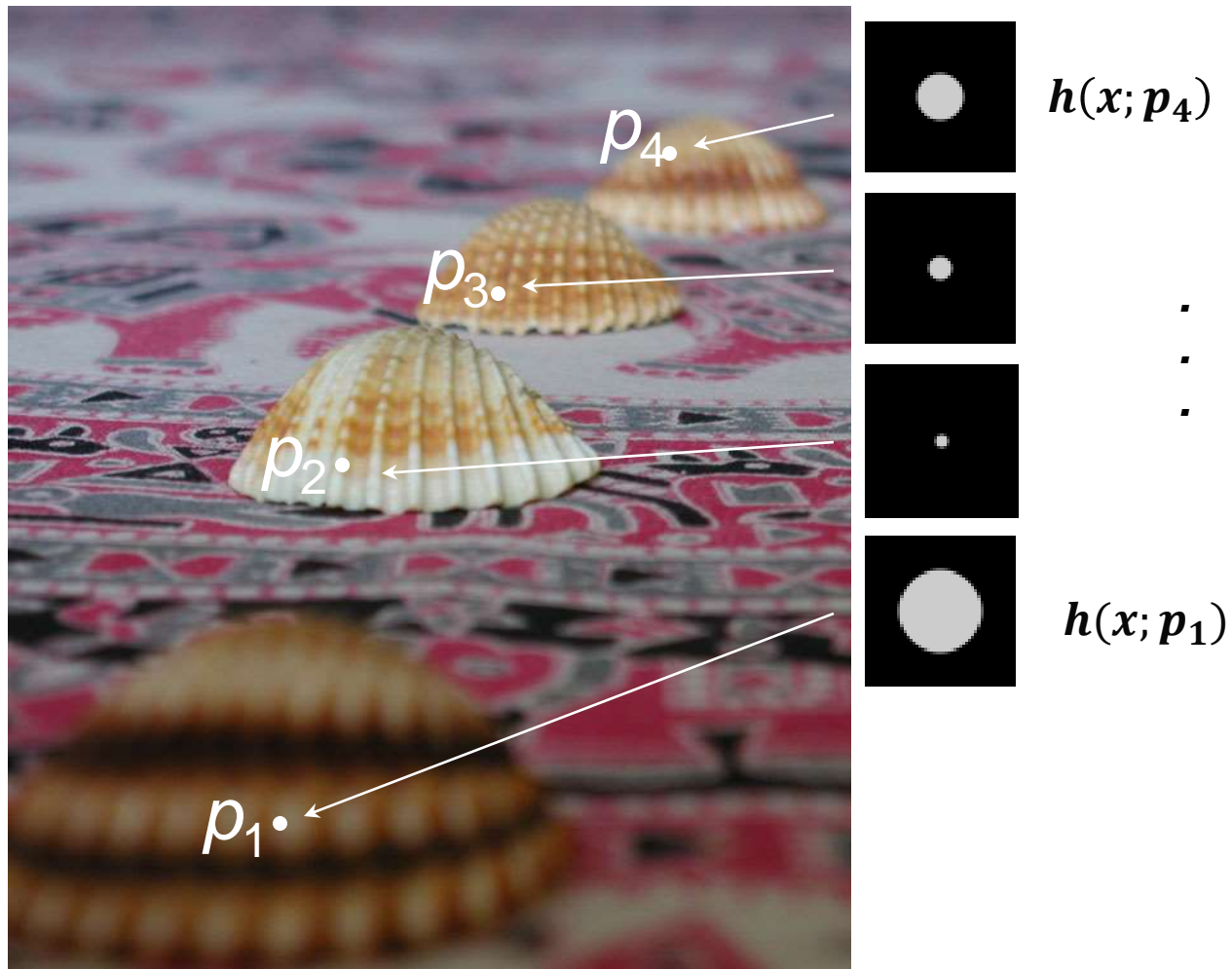
# Object Motion



# Optical aberrations



# Space-variant Out-of-focus Blur



# Approximation of SV Blur

- Patch-wise convolution
  - General SV PSF
  - Locally convolution may not hold
- Parametric model (Blur Basis)
  - More accurate
  - Model may not hold
- Local object motion
  - Segmentation
  - Line blur
- Conversion to space-invariant
  - HW design

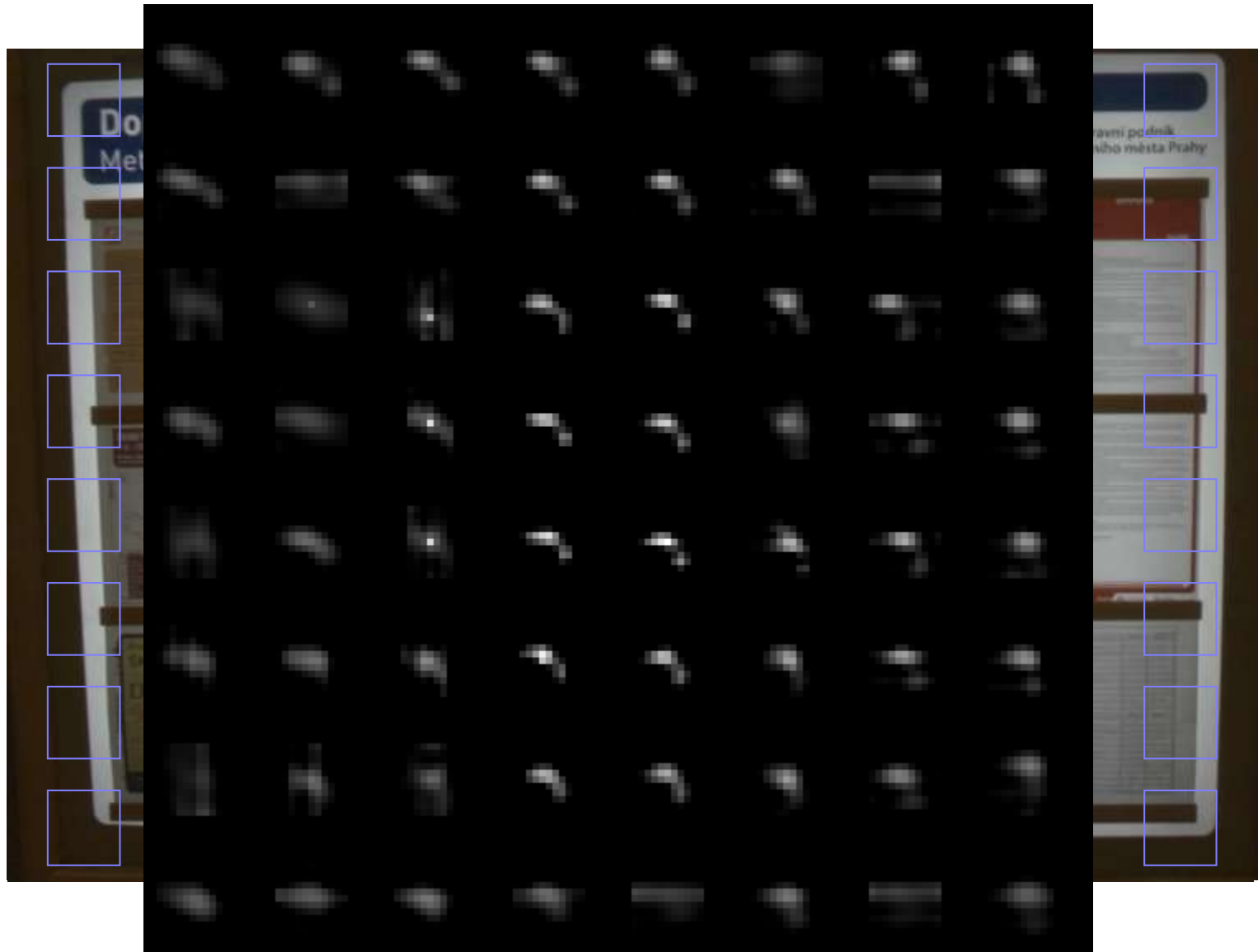
Joshi CVPR08, Sorel ICIP09,  
Ji CVPR12, Sun CVPR15

Whyte CVPR10, Gupta ECCV10,  
Hirsch ICCV11, Zhang NIPS13

Levin NIPS06, Shan ICCV07, Dai CVPR08,  
Chakrabarti CVPR10, Kim CVPR14

Levin SIGGRAPH08

# Patch-wise restoration

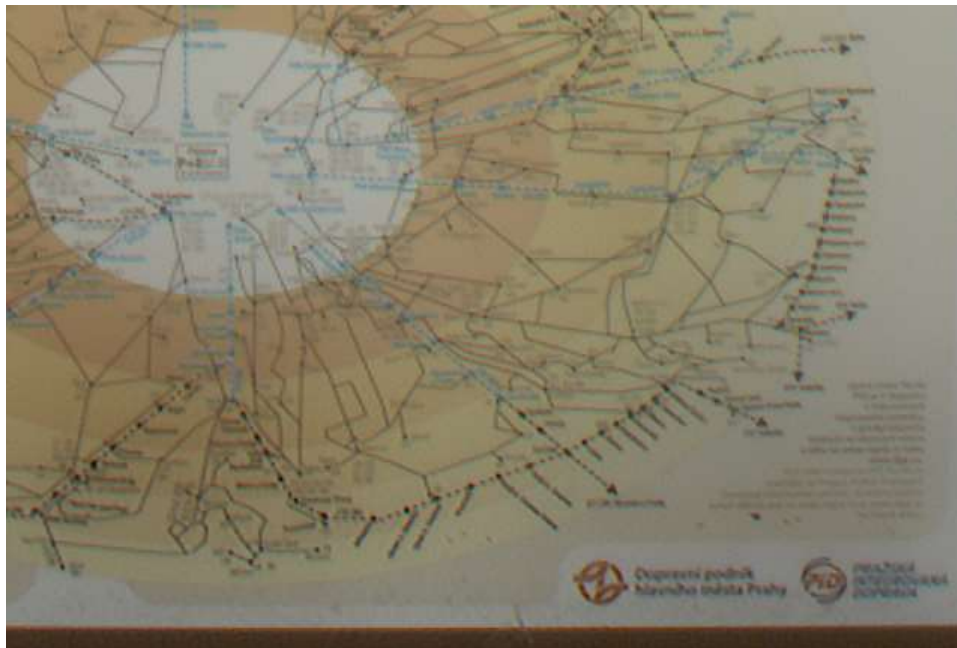




### Jízdenky a kupony MHD

Prague public transport tickets and passes

Typ jízdenky / Ticket type	Dospělý / Adult		Dítě / Child		Senior / Senior	
	10 min	30 min	10 min	30 min	10 min	30 min
10 min	18 Kč	26 Kč	9 Kč	13 Kč	9 Kč	13 Kč
30 min	100 Kč	330 Kč	50 Kč	330 Kč	50 Kč	330 Kč
120 min	500 Kč	1 480 Kč	130 Kč	720 Kč	250 Kč	660 Kč
360 min	670 Kč	1 880 Kč	130 Kč	720 Kč	250 Kč	660 Kč



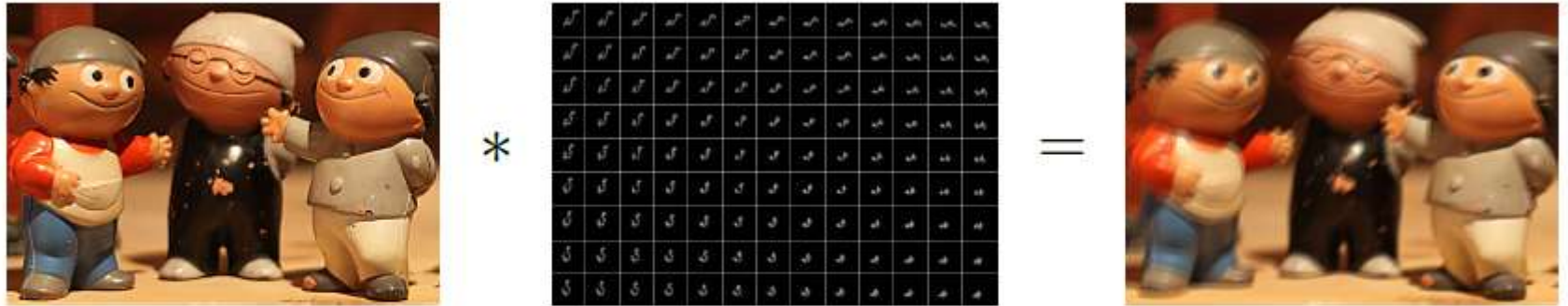
### Jízdenky a kupony MHD

Prague public transport tickets and passes

DRUH JÍZDENKY / KUPONU / Ticket/Pass type	Dospělý / Adult		Dítě / Child		Junior / Junior		Student / Student		Senior / Senior	
	10 min	30 min	10 min	30 min	10 min	30 min	10 min	30 min	10 min	30 min
10 min	18 Kč	26 Kč	9 Kč	13 Kč	18 Kč	26 Kč	18 Kč	26 Kč	9 Kč	13 Kč
30 min	100 Kč	330 Kč	50 Kč	330 Kč	100 Kč	330 Kč	100 Kč	330 Kč	50 Kč	330 Kč
120 min	500 Kč	1 480 Kč	130 Kč	720 Kč	500 Kč	1 480 Kč	500 Kč	1 480 Kč	250 Kč	660 Kč
360 min	670 Kč	1 880 Kč	130 Kč	720 Kč	670 Kč	1 880 Kč	670 Kč	1 880 Kč	250 Kč	660 Kč

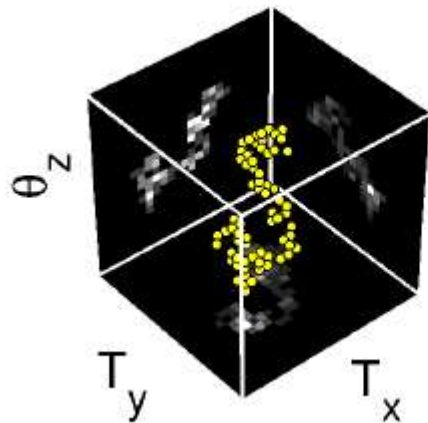


# Parametric Model (Blur Basis)

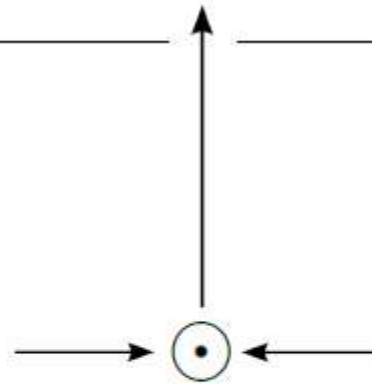


Computation with Efficient Filter Flow

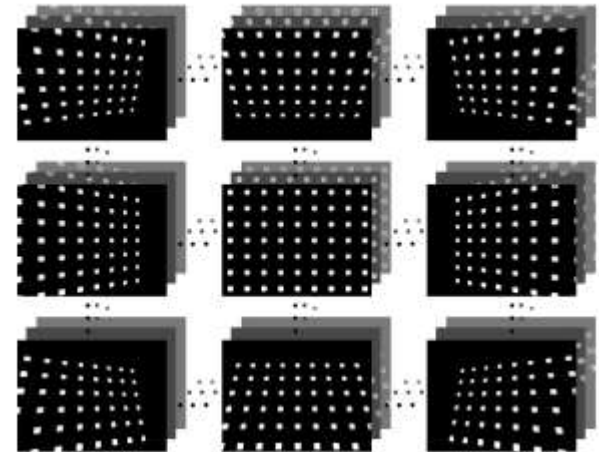
Motion Density Function



$\mu_i$



Point Spread Function Basis

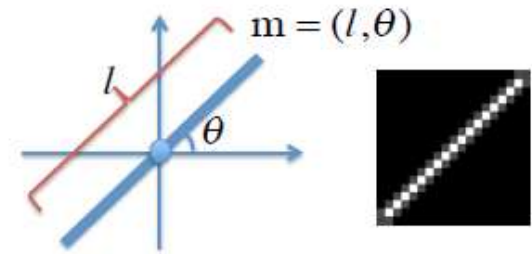


$B_i$

Hirsch ICCV2011

# Object motion

- Estimate locally linear motion blur  
→ line (2 parameters)



- Ignore occlusion
- Consider occlusion

Levin NIPS06  
Chakrabarti CVPR10  
Oliveira TIP14  
Sun CVPR15

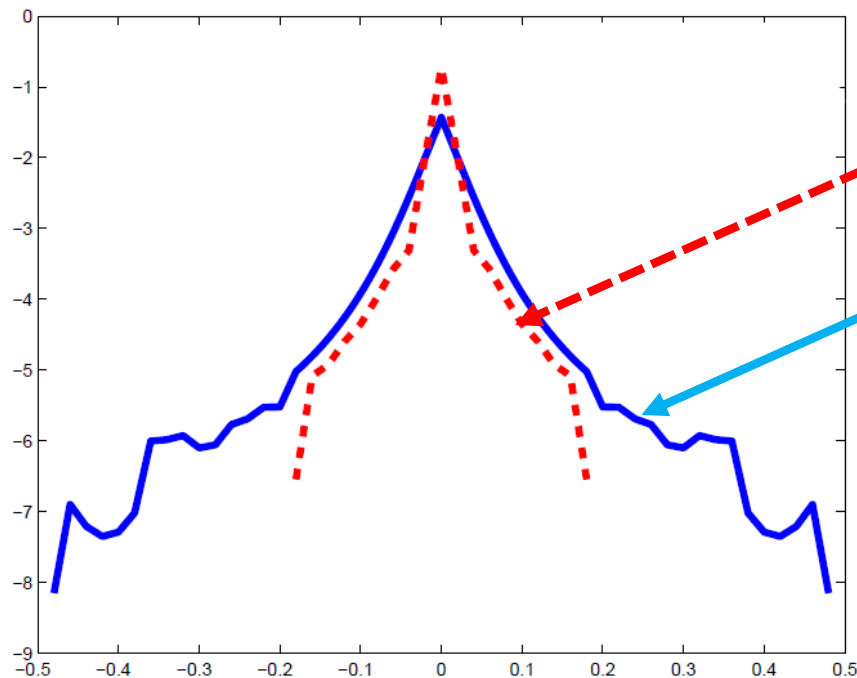
Shan ICCV07  
Dai CVPR08

# Ignore occlusion

Levin NIPS06  
Chakrabarti CVPR10  
Oliveira TIP14

- Local image statistics:

Image autocorrelation increases in the blur direction



Derivatives histogram

Blur direction

Perpendicular direction

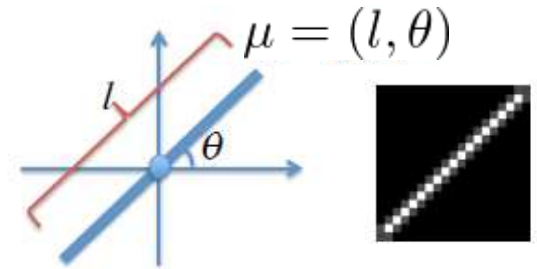
- Correlation
- Histograms
- CNN Sun CVPR15

# Ignore occlusion

Kim CVPR14

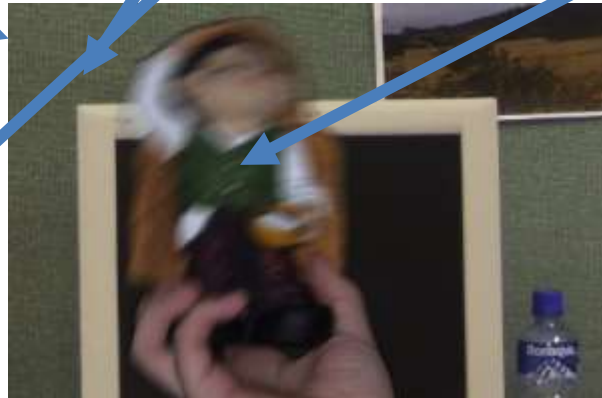
- Blind deconvolution with linear motion constraint

$$\min_{u, \mu} \|(h(\mu) * u) - z\|_1 + Q(u(x)) + R(\mu)$$

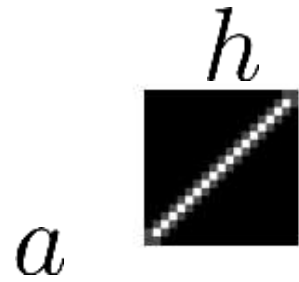
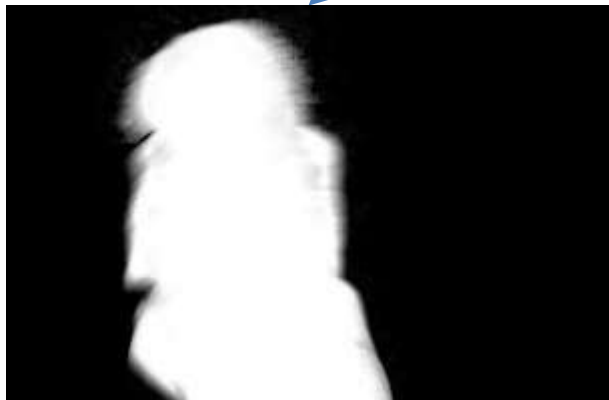


# Occlusion

$$z = (1 - h * a)b + h * f$$



Alpha matting



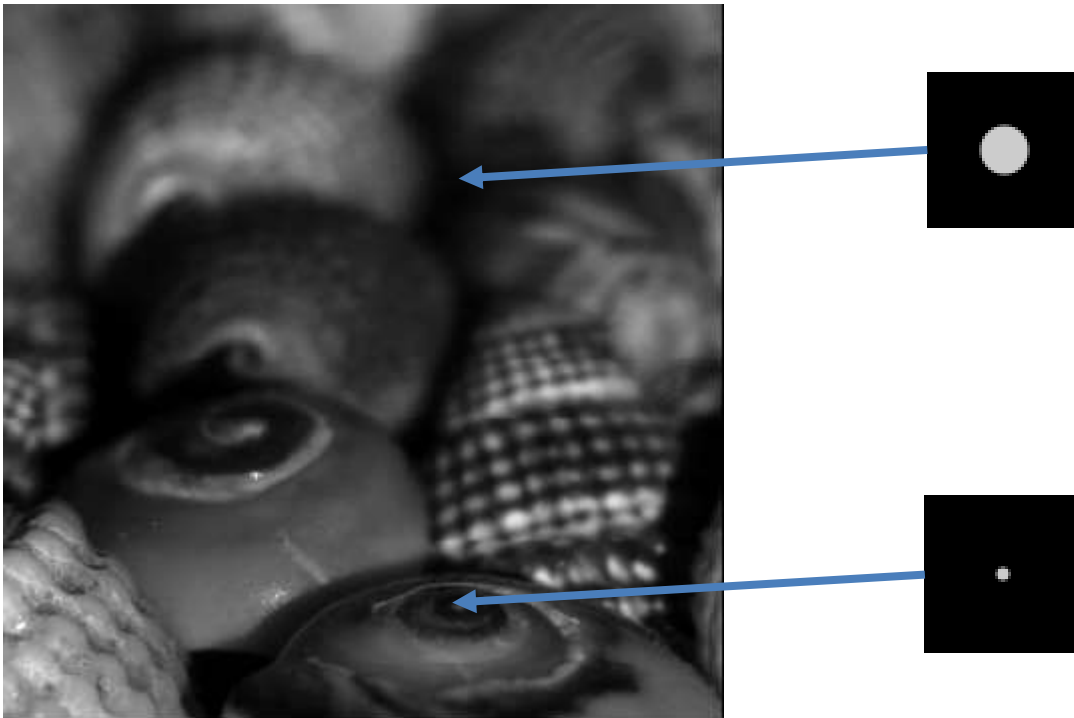
Blind deconvolution

Easier to solve  
result is binary



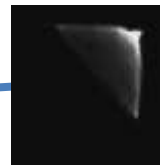
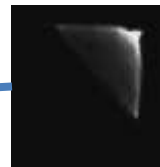
# Conversion to space-invariant

- Wavefront coding – out-of-focus blur



# Conversion to space-invariant

- Wavefront coding



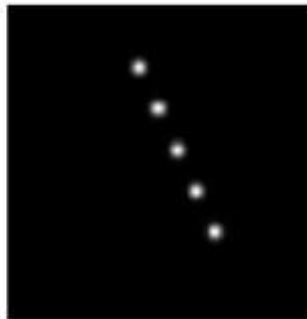
Space-invariant  
non-blind  
deconvolution

# Conversion to space-invariant

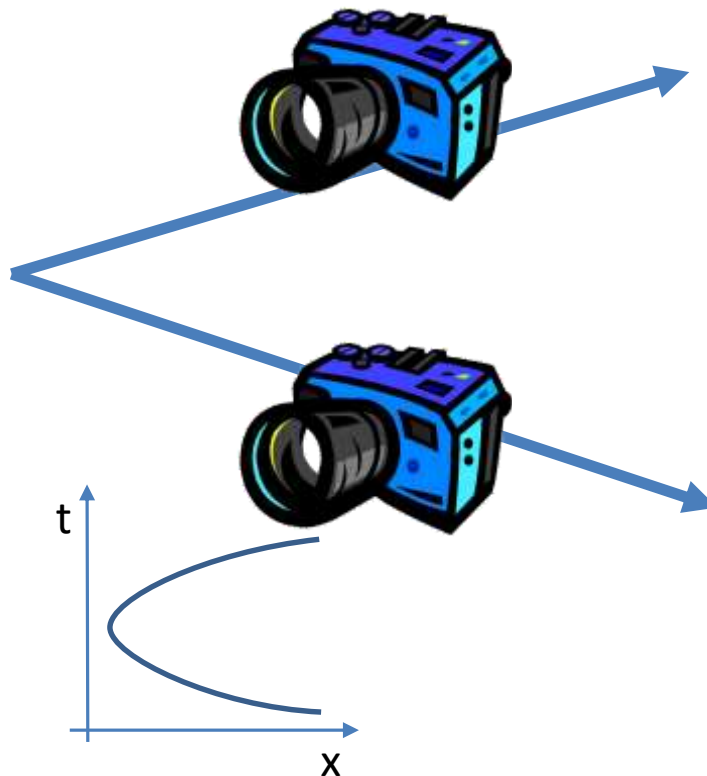
- Motion-invariant photography
  - Space-variant linear motion blur



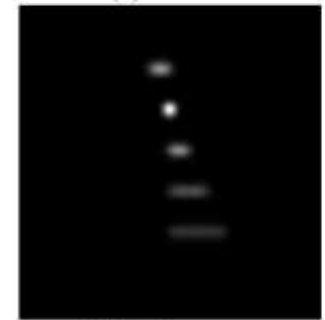
(a) Time 1



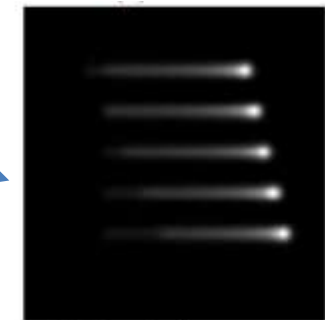
(b) Time 2



Levin SIGGRAPH08



(c) Static camera



(d) Parabolic camera

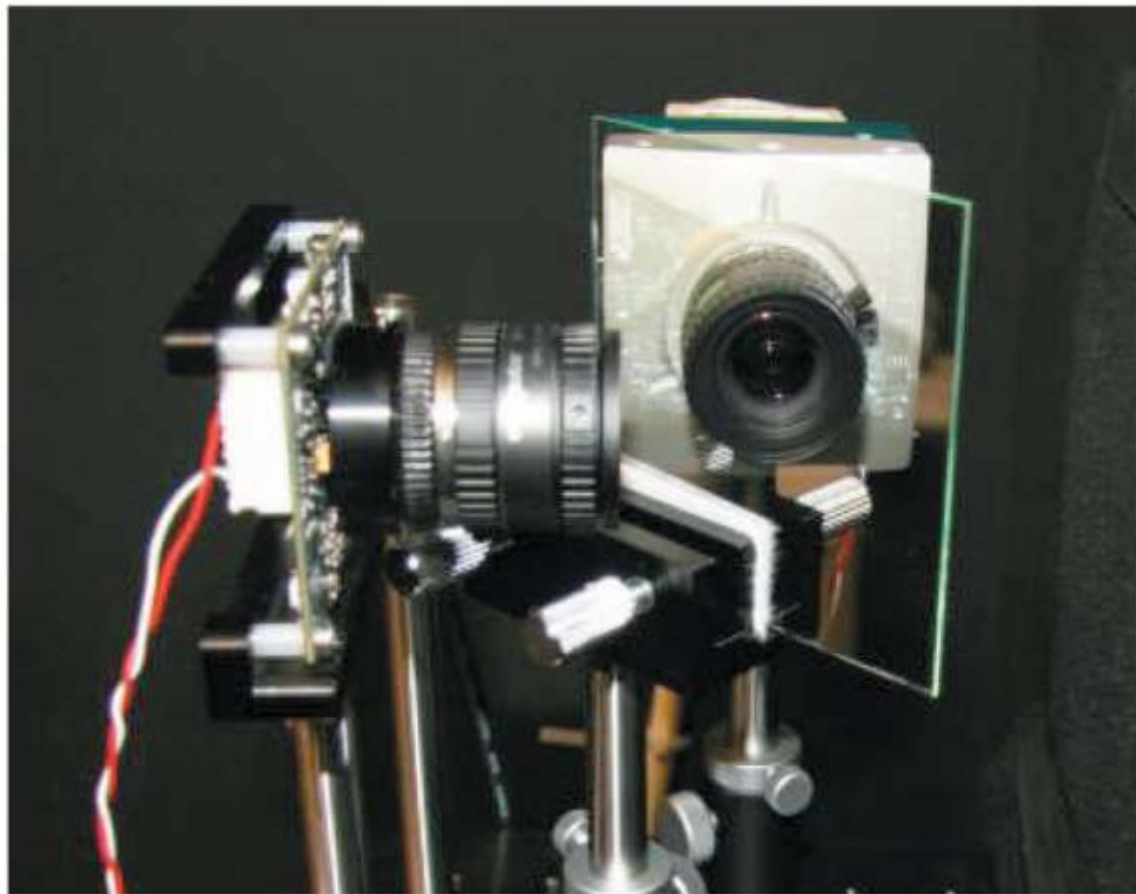


# High-speed cameras

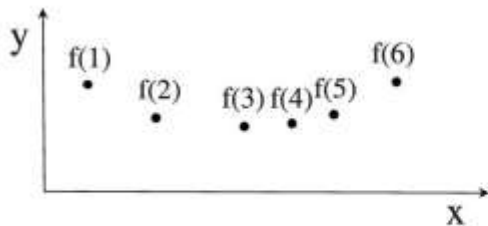
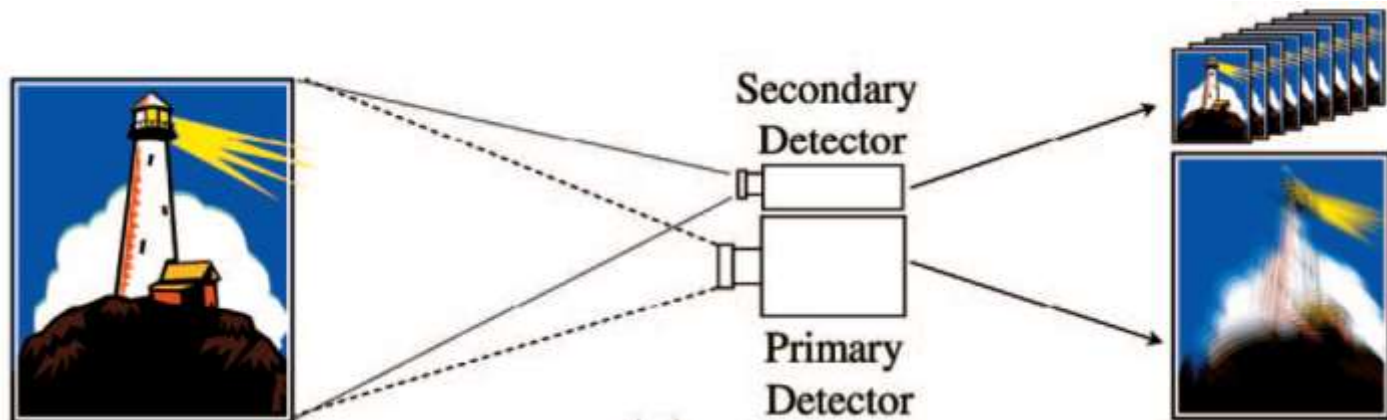
Ben-Ezra PAMI04

Tai PAMI10

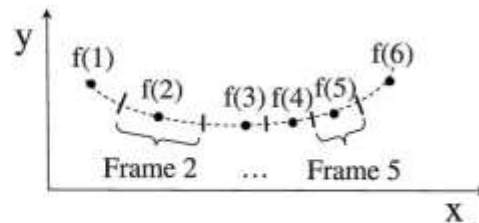
- Low FPS HR camera with high FPS LR camera



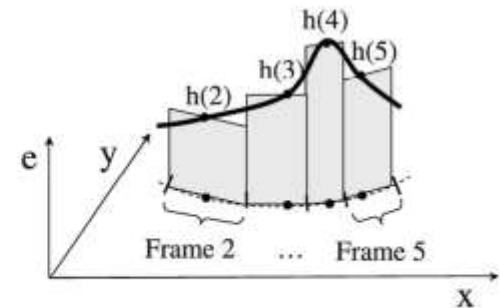
# High-speed camera



Sampled trajectory  
(secondary detector)



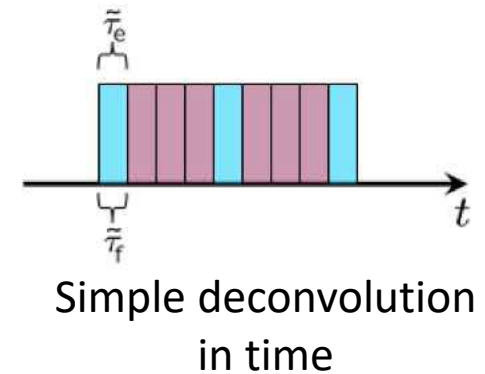
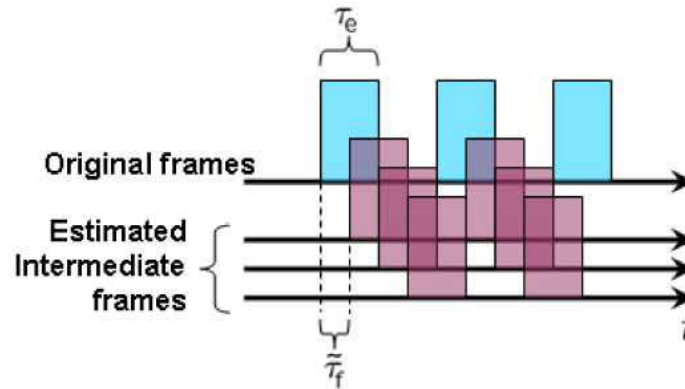
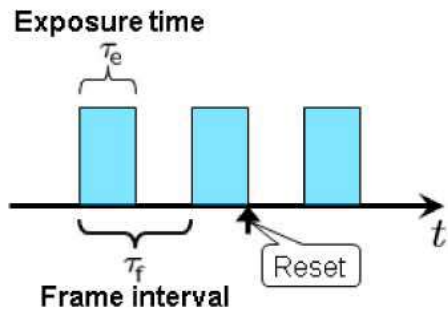
Interpolated trajectory



Estimated PSF

# Space-time Processing

Takeda TIP11

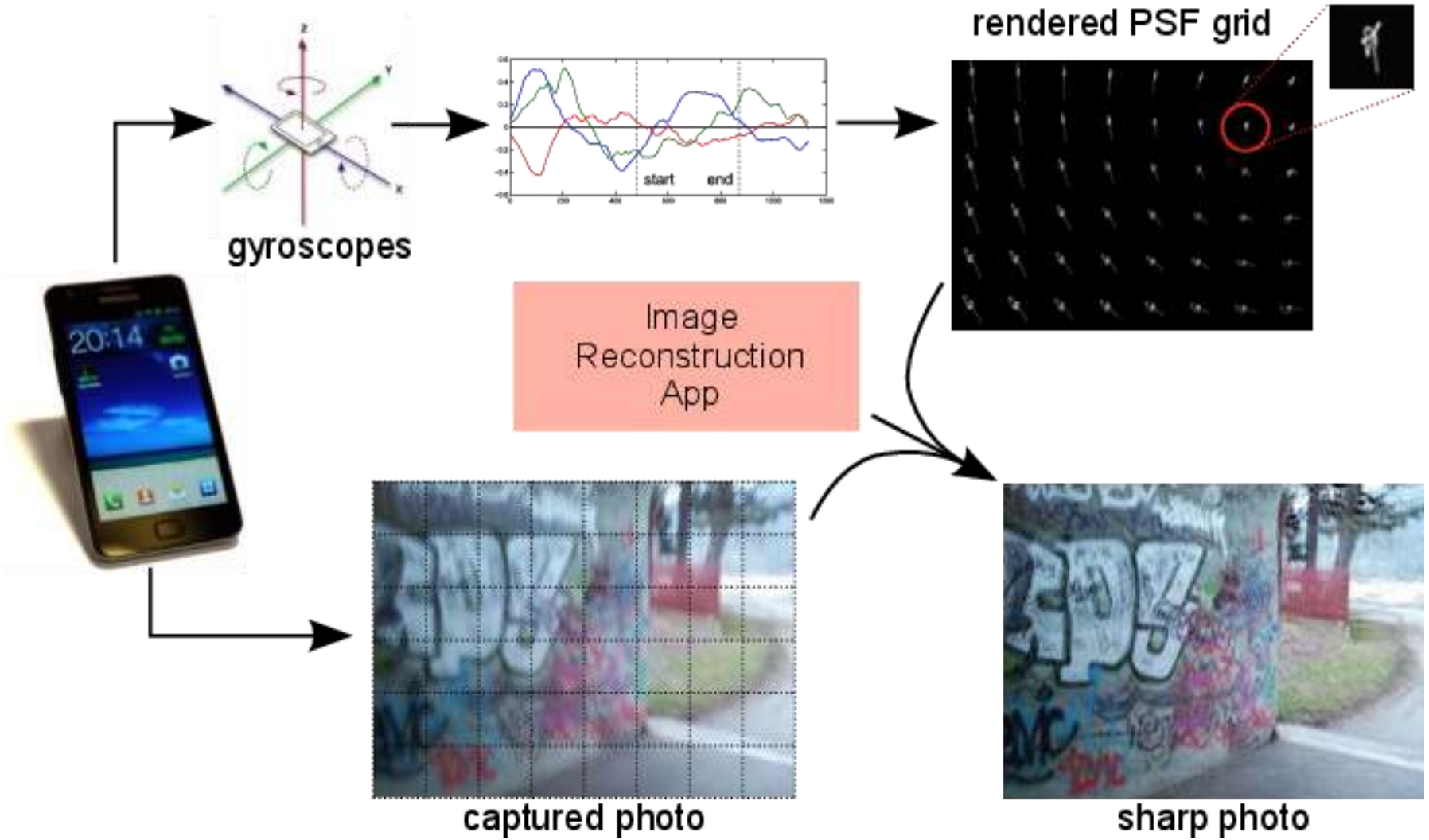


3D steering  
kernel regression



# Gyroscopes

Joshi ACM Trans.Graph.10  
Sindelar JET11  
Levoy



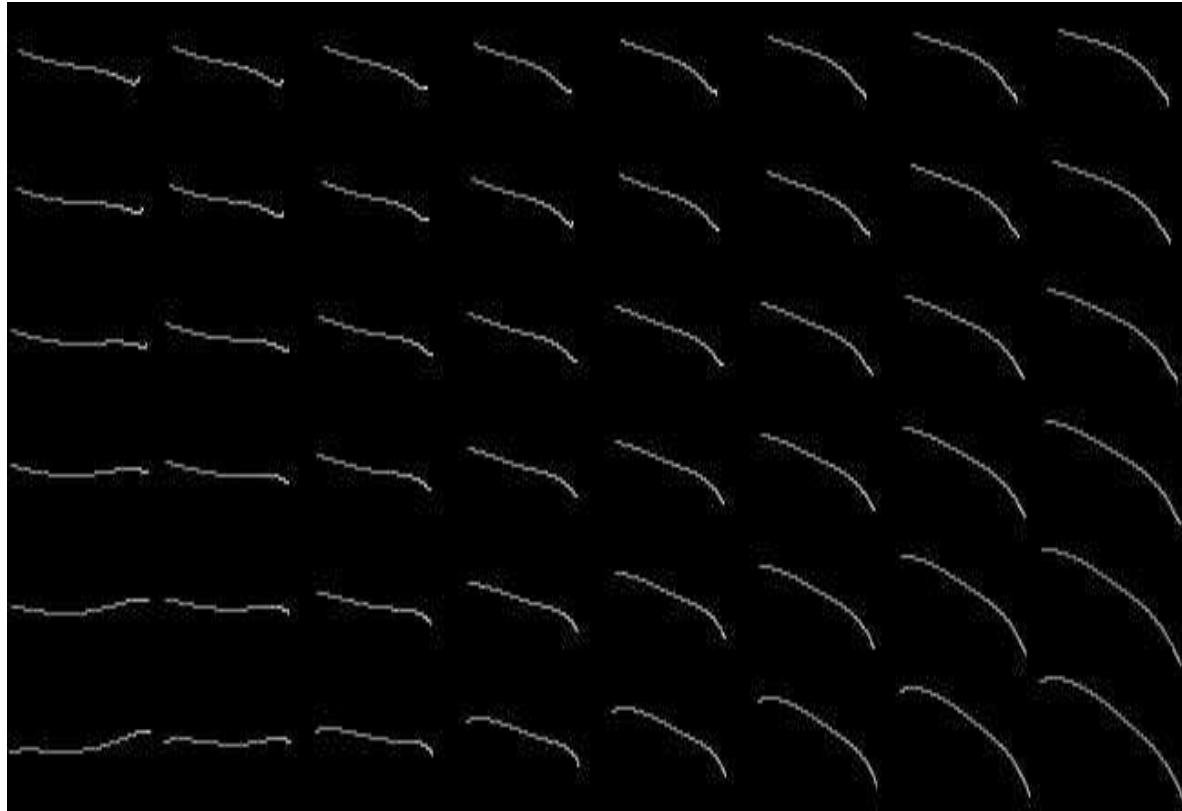


# Acquired blurred image





# Blur estimation





# Patch-wise Deconvolution



# Final Remarks

- MC is far more stable than SC BD.
- How to make SC more robust?
- Can Deep Learning make classical BD methods obsolete?





**Thank You...**